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# Essays in international finance

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**Essays in international finance**

by

**Mohammad Nurul Hasan**

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in partial fulfillment of the requirements for the degree of

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The student author, whose presentation of the scholarship herein was approved by the
program of study committee, is solely responsible for the content of this dissertation.
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Iowa State University

Ames, Iowa

2020

# DEDICATION

I would like to dedicate this thesis to my mother Fatema, and to my wife Farhana without whose support I would not have been able to complete this work. I would also like to thank my friends and family for their loving guidance and financial assistance during the writing of this work.

# TABLE OF CONTENTS

v

# LIST OF TABLES

**Page**

# LIST OF FIGURES

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# ABSTRACT

In this dissertation, I offer two essays on international finance with particular attention to the sovereign debt maturity structure along with the dilution problem and interbank credit crisis that propagates from one country to another. First one is a study on the effect of varying maturity structure on the default probabilities and prices of bonds with and without the dilution problem while the later one develops a financially integrated two-country model with moral hazard and asymmetric information in the banking sector of the economy where endogenously determined interest rates in the economy may cause interbank credit freeze due to overaccumulation of assets and/or a mild productivity shock.

In "Sovereign Debt Maturity Structure and Dilution", I investigate different maturity structure of sovereign bonds; starting from short-term bonds only. I assume that the sovereign can issue only short-term bonds or long-term bonds, and then extended the work with both short- and long-term bonds. Finally, I also investigate what happens to the default probabilities and the prices of bonds if there is a compensation covenant in the sovereign debt contracts for the long-term bondholders to mitigate the dilution problem. The finding is that the maturity structure of sovereign bonds plays an important role in determining the default probabilities and bond prices. Sovereign can improve these by issuing both type of bonds. Moreover, inclusion of the compensation covenant mitigates the dilution problem of sovereign debt in a larger scale when the maturity of long-term bonds is sufficiently high.

In "Interbank Liquidity Crisis and International Contagion", we develop a two-country model with integration in different ways. We investigate integration in terms of labor, deposit, capital mobility both individually and in different combination. Privately known intermediation skills of banks make them heterogeneous and gives rise to the interbank market. When countries are integrated through the banking channel, different corporate lending rates also give rise to different deposit rates and capital supply but the same interbank lending rate. Overaccumulation of assets causes excessive supply of funds during a mild negative productivity shock and creates an interbank market credit freeze in one country which propagates to another country through the interbank lending market.

# CHAPTER 1.   INTRODUCTION

This chapter contains a brief overview of the two essays on international finance in my dissertation.

In chapter 2, "Sovereign Debt Maturity Structure and Dilution", I study the maturity structure of sovereign debt in various combinations- short-term bonds only, long-term bonds only, and both short- and long-term bonds as in benchmark form of Arellano (2008) and in extended form as in Arellano and Ramanarayanan (2012). However, I used the probabilistic maturity structure of bonds as in Leland (1998) and in Chatterjee and Eyigungor (2015). The idea is to compare the default probabilities and bond prices in each scenario. The findings are quite interesting. Sovereign chooses short-term bonds for lower borrowing requirements and long-term bonds for higher borrowing requirements. Issuing long-term bonds provide more borrowing capacity before default is optimal than issuing short-term bonds. Multiple maturity structure improves both default probabilities and bond prices than in case of only short-term bonds or only long-term bonds. I also included the compensation covenant as in Hatchondo et al. (2016) and study the effects on default probabilities and bond prices under cases where sovereign issues only long-term bonds and both short- and long-term bonds. The finding is during low output periods, prices drop less frequently with compensation covenant than without. Compensation covenant reduces dilution problem while also reducing the default probabilities if the maturity of the long-term bond is sufficiently high. Short-term bond price rises with the increase of the maturity of long-term bonds with compensation covenant. From the policy making perspective, sovereign can reduce the default probabilities while keeping

the bond prices high by issuing long-term bonds with sufficiently higher maturity along with short term bonds with compensation covenant.

In chapter 3, "Interbank Liquidity Crisis and International Contagion", we study a two-country model of banking crises in an effort to extend the closed economy model of Boissay et al. (2016). We study the extension in many dimensions- full integration of two countries where depositors can save in any country, labors and mobile, interbank lending is open, and corporate lending market is open; only one of these are open; integration in combinations of these. The finding is the interbank market equilibrium is different for two-country model than the closed economy counterpart only if the interbank lending market is open. Different corporate rates make deposit rates and capital supply in two countries different. However, the main mechanism remains the same. After continuous positive productivity shock, with a mild negative shock interbank credit freeze hits due to overaccumulation of assets by any of the countries. The novel finding is that the interbank credit freeze propagates to another country as well which would have an otherwise functioning interbank lending market. Moral hazard and asymmetric information play important roles in the mechanism. The model is strong in terms of endogeneously determined boom-bust cycles because of a mild shock, not an exogenous big shock as in the financial friction literature. Moreover, the interbank lending rate and corporate rates are endogenously determined, not exogenous as in the sudden stop literature. The findings are interesting. Typical crisis is triggered by a moderate negative productivity shock toward the end of an unusual boom in credit. Endogenously determined interest rates dynamics play a central role to explain sudden interbank credit freeze. A "sail-together" financial integration may go into a "sink-together" interbank credit freeze. From policy making perspective, there are needs of framework for both crisis management and international policy co-ordinations.

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# CHAPTER 2. SOVEREIGN DEBT MATURITY STRUCTURE AND DILUTION

Mohammad Nurul Hasan

Modified from a manuscript to be submitted to the *Journal of International Economics*

## 2.1 Abstract

Maturity structure of debt plays an important role in default probability and the resulting pricing of sovereign bonds. Using a calibrated model this chapter shows that both of these features get improved when the sovereign issues long-term bonds or both short- and long-term bonds instead of issuing only short-term bonds. This chapter also shows that inclusion of the compensation covenant mitigates the dilution problem of sovereign debt in a larger scale when the maturity of long-term bonds is sufficiently high.

**JEL Classification:** F34, H63

**Keywords:** sovereign debt, sovereign default, maturity structure, debt dilution, compensation covenant.

## 2.2 Introduction

An important mechanism of financing the fiscal deficit is issuing sovereign bonds which engendered several policy debates, over the last two decades, including recently debated optimal maturity structure, and debt dilution problem inherent to such sovereign debt. Moreover, it is evident from figure 2.1 (Beers and de Leon-Manlagnit, 2019) that advanced economies are also susceptible to default which adds to the dimension of the sovereign default problem that was supposedly pertinent to only emerging/frontier market economies. Figure 2.1 shows the total sovereign debt in default in US dollar between the period of 1975 and 2018. The figure also depicts that sovereign default problem is not fading out by time rather coming back strongly in new dimensions since nearly 40 percent of sovereign default amount was coming from the advanced economies in the year of 2018. In this chapter I show the linkage between the maturity structure of sovereign debt and the dilution problem. In particular, extending the study of Arellano (2008), I show the sovereign debt contract with only long-term maturity structure and both short- and long-term maturity structure. Default probabilities go down and therefore bond prices go up as we go from bonds with only short-term maturity to bonds with only long-term maturity and finally, to both maturity structure. As a potential solution of the debt dilution problem, I add the compensation covenant in Hatchondo et al. (2016) and find improvement in sovereign bond prices under the framework of Chatterjee and Eyigungor (2015).

If a sovereign keeps increasing debt level then probability of default also increases and sometimes lead to eventual default. A higher default probability is adjusted through reduction in the value of bonds- a debt dilution faced by the sovereign. Debt dilution problem arises because current government cannot control the bond issuances by the future governments. It is not possible to restrict a sovereign from issuing bonds since doing so may lead to restricting the sovereign from consumption. Therefore, a covenant of "no
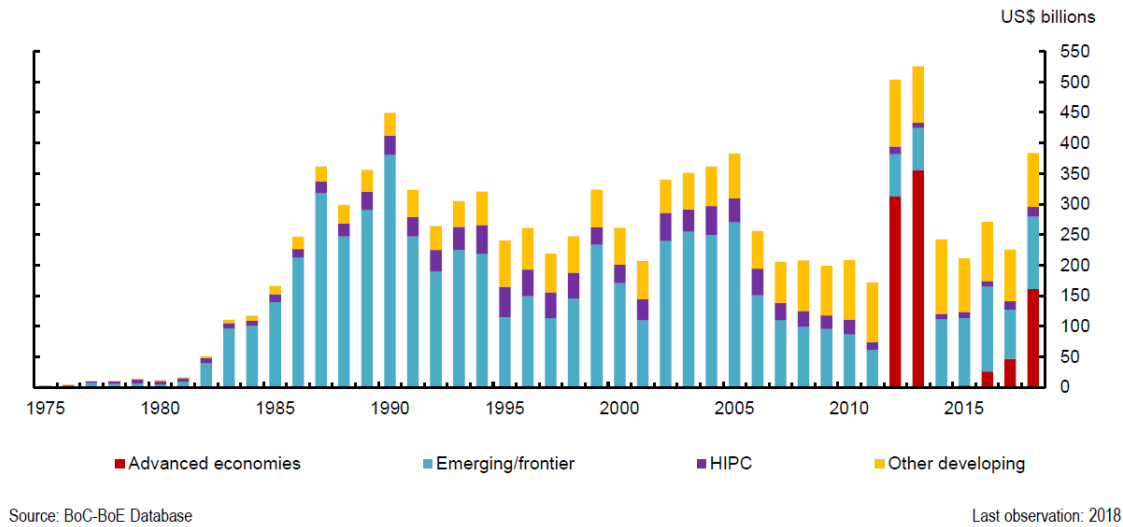
Figure 2.1: Sovereign Debt in Default

further issuance of bonds" is never a feasible idea. However, any additional bond issued by the sovereign results in higher risk of default anticipated by the rational investors and therefore investors offer lower price for the bonds. Eventually, the sovereign faces a dilution in the value of their issued bonds in the market. A possible way of reducing the debt dilution problem is to shorten the maturity of the bonds. However, shortening the maturity of bonds would increase the rollover risk for the sovereign, i.e., the risk of higher borrowing cost. There is a trade-off between the short-term debt which has a relative incentive benefit and the long-term debt which has a hedging benefit. This trade-off of the maturity structure is reflected through the price functions. Therefore, there should be a suitable solution for the optimal maturity structure and the debt dilution problem.

Government bonds are issued in the primary market and traded in the secondary market. The price is determined based on the default premium coming from the default risk of the sovereign at the time the bond is issued in the primary market. In the secondary market, the price reflects the current situation of the sovereign. Issuing more bonds increases the default risk of the sovereign and therefore default premium increases

to reduce the price of the bond. The senior bondholders (who purchased bonds in earlier periods) are affected by the borrowing decision of the sovereign since the price at which they can sell the bonds they hold in the secondary market is lower than what would have been without more borrowing by the bond issuing sovereign. Even if the bondholders hold their bonds until maturity, they face an increased default risk due to increase in debt level through issuance of more bonds by the sovereign. Only the new bondholders get the compensation of the higher default risk through lower bond price. This requires to arrange a compensation package for the existing bondholders before any new issuance of sovereign bonds.

Several authors propose varying solutions to mitigate the debt dilution problem. While Hatchondo et al. (2016) propose imposing a maximum limit on either total foreign debt or debt-to-GDP ratio, Chatterjee and Eyigungor (2015) corroborate the idea of seniority of sovereign debt creditors. The idea proposed by Hatchondo et al. (2016) is supposed to reduce the probability of default and ensure the existing bondholders a compensation in case the sovereign wants to issue more bonds. Bond covenant can include the clause of compensation equal to the differential of the observed price and the price at which the bonds would be traded without new bond issues. To circumvent the difficulty of finding the counterfactual price, the authors study two covenants which set a penalty if the sovereign chooses a debt level above a threshold or borrows at bond prices below a threshold. However, a differential compensation is not enough for the existing bondholders when the sovereign defaults and all the creditors, both earlier and later, get repaid at the same proportion during the settlement. Moreover, Hatchondo et al. (2016) use debt-to-GDP ratio as a measure of debt level which requires continuous adjustment of sovereign debt due to the change in GDP, if the ratio is close to the threshold.

To avoid the problem of continuous adjustment of sovereign debt, Chatterjee and Eyigungor (2015) propose a relative seniority arrangement where all the creditors lose

an equal amount of outstanding debt but get paid based on pre-decided seniority for the remaining portion of debt in case of reorganization after default by the borrowing sovereign. This is for the resolution of dispute among the creditors under absolute seniority arrangement. A seniority arrangement specifies the allocation of repayments after default and therefore, such an arrangement is not a suitable candidate for the solution of the sovereign debt dilution problem. Even with a seniority arrangement, debt dilution problem exists. For instance, an investor A may buy a long-term sovereign bond and would like to sell it before the maturity. If the sovereign issues further long-term bonds then the default risk increases for the sovereign and the price goes down in the secondary market to adjust this risk. If the investor A is still holding the previously issued bond then he faces the downward price movement when selling the bond before maturity. Now, a seniority rule may act as an insurance for the investor for getting the investment back after the default by the sovereign. However, such arrangement fails to even reduce the problem of downward price movement faced by the investors like A who intend to sell the security before the maturity. Therefore, seniority arrangement cannot fully eliminate the debt dilution when the default probability is increased because of new borrowings. Bizer and DeMarzo (1992) also observe that a seniority arrangement is not sufficient to eliminate debt dilution when default probability is increased through new borrowings.

Following Arellano (2008), this chapter initiates with the basic model of sovereign debt of only one period, i.e., short-term bond. Then, I study the model with only long-term bond followed by the model with multiple maturity structure which includes both the short- and long-term bonds. I also include the compensation covenant in Hatchondo et al. (2016) for both long-term bonds contract and contracts with short- and long-term bonds as a solution of the debt dilution problem. However, I follow the model structure of Chatterjee and Eyigungor (2015) with a probabilistic maturity of the bonds. In the appendix, I show the sovereign debt model with settlement of defaulted bonds where

settlement happens in one of the later periods when the sovereign gets back to the international financial market after an exclusion due to the default decision on outstanding debt. Finally, I also explore the competing solutions of the debt dilution problem, e.g., debt threshold covenant, price threshold covenant, and income-contingent debt.

The chapter proceeds from here with the literature review of sovereign debt maturity structure and the dilution problem in section 2.3. In section 2.4, there are basic models of only short-term sovereign bonds as in Arellano (2008), long-term bonds in Chatterjee and Eyigungor (2015) framework, and bonds with both maturity structure as in Arellano and Ramanarayanan (2012). Section 2.5 shows the models without the debt dilution problem while introducing compensation covenant to the existing bondholders with both long-term bonds and maturity structure of short- and long-term bonds. Section 2.6 presents the comparative analyses of all the models used for different maturity structure of long-term bonds. Section 2.7 concludes the chapter .

## 2.3  Literature Review

Sovereign debt has an extensive literature covering but not limited to existence and uniqueness of equilibrium, strategic structure of the debt market, contract and maturity choice, settlement following default, restructuring without default, reputation and partial default, debt dilution and added covenants, fiscal rules and default, default cost, and contagion and correlated defaults. The seminal paper in sovereign debt literature is Eaton and Gersovitz (1981) where the authors introduce the sovereign debt commitment problem and default decision of the sovereign in comparison to repayment based on relative welfare on either decisions. There is a substantial body of work in sovereign debt literature based on the Eaton and Gersovitz (1981) framework. See Aguiar et al. (2016) for a detailed quantitative literature on diverse issues of sovereign debt.

Arellano (2008), with a one period model, shows how default is more likely in recessions using business cycles data of Argentina. However, while the defaulting sovereign and the creditors resolve through settlements in most of the cases, this paper assumes that the sovereign never pay back the defaulted debt. Later, Arellano and Ramanarayanan (2012) study on the emerging markets term structure of interest rate spreads and the maturity composition of government debt and show that the sovereign has more incentive to repay in case of short-term debt whereas long-term debt is a hedge against future fluctuations in spread. Understanding the emerging market sovereign debt maturity structure requires comprehending this trade-off between the incentive benefits of short-term debt and hedging benefits of long-term debt. Broner et al. (2013) document that shorter-maturity debt is issued in crisis periods while long-term debt is issued in normal times by the emerging market sovereigns.

Although the solution of debt dilution has got attention recently, the literature on sovereign dent dilution is intensive. Borensztein et al. (2006) observe that emerging-market borrowers face a lengthy and costly period of restructuring due to debt dilution. The possibility of dilution induces the sovereign borrowers to opt for short-term bonds (Kletzer, 1984; Sachs and Cohen, 1982) since such bonds are difficult to dilute or debt those cannot be restructured easily to raise the cost of default and lower the likelihood of default (Dooley, 2000). The short-term debt introduces the rollover crisis driven by lenders' confidence (Giavazzi and Pagano, 1990; Cole et al., 1996), and hard-to-dilute bonds worsen the crisis inefficiently (Bolton and Jeanne, 2009).

Absolute priority rule in repayments of bondholders is explicitly studied in sovereign debt literature (Borensztein et al., 2006; Bolton and Skeel Jr, 2004; Bolton and Jeanne, 2009; Gelpern, 2004). However, absolute seniority is not a panacea of debt dilution problem and therefore there should be a relative seniority rule (Chatterjee and Eyigungor, 2015). There

may also be a compensation package for the existing bondholders before issuing any new bonds by the sovereign (Hatchondo et al., 2016).

## 2.4 The Basic Models

### 2.4.1 Sovereign Debt with Short-term Bonds

We have a small open economy where the sovereign government decides on both consumption and savings/borrowing on behalf of the domestic residents who face fluctuating endowment stream. The sovereign faces the given world risk-free interest rate and cannot change it through any decisions. The sovereign can borrow from the credit market where the lenders are risk-neutral. Sovereign debt contracts are not enforceable in the international financial market and therefore, a sovereign can take a default decision at any time on the outstanding debt. In case of default, the sovereign is assumed to be excluded temporarily from the international credit market and to incur output costs directly. Sovereign bond price available to the sovereign make creditors break even in expected value and reflect the default event likelihood.

The paper proceeds from here by describing the endowment and preference structure of the sovereign first. Then, we give description of the financial market and events timing. Afterwards, we describe both the sovereign's and the lenders' problem followed by the recursive equilibrium.

The economy has identical, risk averse households who have preferences given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_t), \ \ 0 < \beta < 1. \tag{2.1}$$

where, $c$ is the consumption, $\beta$ is the discount factor, and $u(\cdot)$ is increasing and strictly concave. $y$ is the tradable good with a stochastic stream and received by the households. The output shock is a Markov process with a transition function $f(y', y)$ with a compact

support. There is a lump sum transfer of goods from the government to the households. The former is benevolent with an objective of maximizing the utility of the later.

The government can sell one-period discount bonds $b'$ at price $q(y, b')$ in the international financial markets and decides, at the end of the period, whether to default or repay on its outstanding debt. The default probability depends on both the aggregate shock $y$, and the size of the bond $b'$. Therefore, the bond price function $q(y, b')$ also depends on both of these two and is endogenous to the default incentive of the government. Purchasing a discount bond with price $q(y, b')$, where $b' \geq 0$, means saving $q(y, b')b'$ units of goods in the current period to receive next period $b'$ units of goods. Purchasing a discount bond with price $q(y, b')$, where $b' \leq 0$, means receiving $-q(y, b')b'$ units of goods in the current period to deliver $b'$ units of goods the next period, conditional on not declaring default. The households receive all the proceeds of the international credit operations by the government in a lump sum way.

Being benevolent, the government, with an effort to smooth consumption, uses borrowing form the international credit market effectively. $y$ induces an idiosyncratic income uncertainty, and the asset markets are incomplete due to two reasons: (i) an endogenous default risk, (ii) an uncertain set of available assets. The set of available bonds pay a state and time invariant amount which cannot insure away the idiosyncratic income uncertainty.

The cost of default has two components: direct output cost, $\phi(y)$ and exclusion from the international financial market. This specification follows the results of defaulting from recent events, i.e., lower aggregate output and a temporary loss of access to the international financial market. With this specification, all outstanding debts get erased from the budget constraint of the government and it is no longer allowed to participate in the international financial market, i.e., the government cannot save or borrow. Following a

default, there is a stochastic number of period for the government in financial autarky and reenters the markets with an exogenous probability $\xi$.

Following is the government resource constraint of a small open economy:

$$c \leq \begin{cases} y - q(y, b')b' + b & \text{Repayment} \\ y - \phi(y) & \text{Default} \end{cases} \tag{2.2}$$

The international credit market has a constant interest rate, $r^* > 0$, at which investors can lend or borrow as much as required. Investors can observe the income level every period and therefore, they possess perfect information about the endowment process of the economy. Foreign creditors are risk-neutral, i.e., in every bond contract offered they break even in expected value. Every period lenders choose loans $b'$ to maximize expected profits $\pi$, taking prices as given:

$$\pi = q(y, b')b' - \frac{1 - d(y, b')}{1 + r^*}b' \tag{2.3}$$

where $d(y, b')$ is the probability of default which depends on the level of income, $y$ and level of foreign asset holdings, $b'$.

The probability of default, $d(y, b')$ is zero for positive level of foreign asset holdings, $b' \geq 0$. In this case, the price of a bond is equal to the opportunity cost of the creditors. If the level of foreign asset holdings in negative, $b' < 0$, then the equilibrium bond price equals to the opportunity cost after risk-adjustment. This bond prices are, therefore, required to satisfy

$$q(y, b') = \frac{1 - d(y, b')}{1 + r^*} \tag{2.4}$$

Following is the the timing of events within each period. The government, foreign creditors, and households act sequentially. The government enters each period with asset level $b$ to repay at the end of the period. The income shock $y$ is the observed by the government, who decides whether to default or repay its outstanding debt obligations at the

end of the period. If the government decides to repay, then chooses $b'$ subject to resource constraint, given the bond price schedule $q(y, b')$. Otherwise, the government defaults on its debt and exits the international financial market for a stochastic period of time. In case of repayment by the government, creditors choose $b'$, taking $q$ as given. Following these events, consumption $c$ takes place by the households. Consumption includes the endowment and the government transfer from the international financial market credit operations.

The economy is characterized by a recursive equilibrium where the government's policy function $b'$, bonds' price function $q$, and consumers' policy functions $c$ determine the equilibrium, given aggregate states $s = (y, b)$. The government chooses whether to repay or default, given initial foreign assets $b$ and after observing the income shock $y$. With the choice of repayment of debt obligations, the government then decides the new level of foreign assets $b'$, given the price of new borrowing $q(y, b')$, that depends on the states $y$ and the choice of $b'$.

Let $V^o(y, b)$ be the lifetime utility of a sovereign that enters a period with $V^o(y, b)$ and maintains a good standing (not in default state) and $V^d(y)$ be the lifetime utility of a sovereign in default.

Let $V^o(y, b)$ be the sovereign's lifetime utility with good standing (not in default) while entering a period with $(y, b)$ and $V^d(y)$ be the sovereign's lifetime utility in default. When the government decides to default, the economy goes into a financial autarky for a short period of time and income falls by $\phi(y)$. Then the value of default is given by,

$$V^d(y) = u(y - \phi(y)) + \beta E_{\{y'|y\}}[(1 - \xi)V^d(y') + \xi V^o(y', 0)] \tag{2.5}$$

Where, $\xi$ is the probability of reentry in the international financial market.

The payoff from repaying the debt and remaining in the contract, denoted $V^c(y, b)$, is given by

$$V^c(y, b) = \max_{b' \in \mathbb{B}}[u(y - q(y, b')b' + b) + \beta E_{\{y'|y\}}V^o(y', b')] \tag{2.6}$$

The government solves the utility maximization problem by choosing the optimal level of new debt $b'$ in each period. The expected value of remaining in the contract incorporates the fact that default choice could be made by the government at any time in the future. To prevent any Ponzi schemes, there is a lower bound on the level of new debt that government may choose, $b' \geq -Z$ which is not binding in equilibrium otherwise.

Finally, given the default option, $V^o(y, b)$ satisfies

$$V^o(y, b) = \max\{V^c(y, b), V^d(y)\} \tag{2.7}$$

The default policy of the government can be characterized by repayment sets and default sets. Let $R(b)$ be the set of income levels for which the optimal decision is repayment when assets are $b$, such that

$$R(b) = \{y \in Y : v^c(y, b) \geq v^d(y)\}$$

and let $D(b) = \tilde{R}(b)$ be the set of income levels for which the optimal decision is default for assets level of $b$:

$$D(b) = \{y \in Y : v^c(y, b) < v^d(y)\}$$

The equilibrium bond price function $q(y, b')$ should correctly assess the probability of default by the government and therefore, needs to be consistent with the optimization of the government and with the expected zero profits for the lenders. Default probabilities $d(y, b')$ and default sets $D(b')$ are therefore related in the following way:

$$d(y, b') = \int_{D(b')} f(y', y)dy' \tag{2.8}$$

The equilibrium default probabilities $d(y, b')$ are equal to zero when there are empty default sets, $D(b') = \emptyset$ because the optimal choice is to repay for all realizations of the endowment shocks, with assets $b'$. Default probabilities $d(y, b')$ are equal to one when $D(b') = Y$ because for any realization of the endowment shocks it is optimal to default.

Definition 1: *The definition of the recursive equilibrium of the economy is given by a set of policy functions (i) government's asset holdings $b'(s)$, default sets $D(b)$, and repayment sets $R(b)$, (ii) consumption $c(s)$, and the price function for bonds $q(y, b')$ such that*
1. *Given the bond price function $q(y, b')$, the government's policy functions $b'(s)$, default sets $D(b)$, and repayment sets $R(b)$ satisfy the government optimization problem.*
2. *Given the government policies, households' consumption $c(s)$ satisfies the resource constraint.*
3. *Bond prices $q(y, b')$ are consistent with both the default probabilities of the government and the expected zero profits by the creditors.*

### 2.4.2   Sovereign Debt with Long-term Bonds

This section differs from the previous one in terms of the maturity of the debt contracts. Here, long-term debt contracts mature probabilistically (Leland, 1998; Chatterjee and Eyigungor, 2015). Specifically, there is a probability $\lambda$ that determines the maturity of each unit of outstanding debt next period. In case, the debt does not mature, there is an associated coupon payment $z$. Debt service requires to pay $\lambda b$ for matured portion of debt and $(1 - \lambda)zb$ for the periodic coupon payment for the portion of debt that is not matured. Therefore, total debt service is $[\lambda + (1 - \lambda)z]b$. Debt level of the next period is denoted $b'$ where the debt that is not matured in the current period is $(1 - \lambda)b$. Hence, total new issue of bonds is $(1 - \lambda)b - b'$ and with bond price $q(y, b')$, total credit funding for the consumption is $q(y, b')[(1 - \lambda)b - b']$.

Equation (1) will still hold since it does not depend on the maturity structure of bonds.

Government resource constraint for issuing long-term bonds only is as follows:

$$c \leq \begin{cases} y + q(y, b')[(1 - \lambda)b - b'] + [\lambda + (1 - \lambda)z]b & \text{Repayment} \\ y - \phi(y) & \text{Default} \end{cases} \qquad (2.9)$$

The value function of a sovereign in a state of default is:

$$V^d(y) = u(y - \phi(y)) + \beta E_{\{y'|y\}}[(1 - \xi)V^d(y') + \xi V^o(y', 0)] \qquad (2.10)$$

Let $V^c(y, b)$ be the payoff from repaying the debt which is given by

$$V^c(y, b) = \max_{\{b', c\}}[u(c) + \beta E_{\{y'|y\}} V^o(y', b')] \qquad (2.11)$$

Finally,

$$V^o(y, b) = \max\{V^c(y, b), V^d(y)\} \qquad (2.12)$$

The default policy of the government can be characterized by repayment sets and default sets. Let $R(b)$ be the set of income levels for which the optimal decision is repayment when long-term debt is $b$, such that

$$R(b) = \{y \in Y : v^c(y, b) \geq v^d(y)\}$$

and let $D(b) = \tilde{R}(b)$ be the set of income levels for which the optimal decision is default for the debt level of $b$:

$$D(b) = \{y \in Y : v^c(y, b) \leq v^d(y)\}$$

Under competition, a unit bond price satisfies the pricing equations as follows:

$$q(y, b') = \frac{1}{1 + r^*} \int_{R(b')} [\lambda + (1 - \lambda)\{z + q(y', \tilde{b}')\}]f(y', y)dy' \qquad (2.13)$$

where,

$$b' = \tilde{b}(y, b)$$

### 2.4.3  Sovereign Debt with Multiple Maturity Structure

In this section I assume that the sovereign can issue both short- and long-term bonds as in Arellano and Ramanarayanan (2012). The sovereign now start a period with $b_S$ level of short-term bonds and $b_L$ level of long-term bonds. There are two prices now- $q_S(y, b'_S, b'_L)$ for short-term bonds and $q_L(y, b'_S, b'_L)$ for long-term bonds. The price of short-term bonds does not only depend on the its own level of debt level chosen for the next period but also on the long-term debt level $b'_L$. This is because with each unit of bond of any maturity structure, the default probability of the sovereign goes up and the prices of both short- and long-term bonds depend on the default probability. Revenue collected from issuing short-term debt is $-q_S(y, b'_S, b'_L)b'_S$ and from long-term debt is $q_L(y, b'_S, b'_L)[(1-\lambda)b_L - b'_L]$.

Government resource constraint for issuing both short- and long-term bonds is as follows:

$$c \le \begin{cases} y - q_S(\cdot)b'_S + q_L(\cdot)[(1-\lambda)b_L - b'_L] + b_S + [\lambda + (1-\lambda)z]b_L & \text{Repayment} \\ y - \phi(y) & \text{Default} \end{cases} \tag{2.14}$$

where, $q_S(\cdot) = q_S(y, b'_S, b'_L)$ and $q_L(\cdot) = q_L(y, b'_S, b'_L)$.

The value of default is given by:

$$V^d(y) = u(y - \phi(y)) + \beta E_{\{y'|y\}}[(1-\xi)V^d(y') + \xi V^o(y', 0, 0)] \tag{2.15}$$

Let $V^c(y, b_S, b_L)$ be the payoff of the sovereign from repaying the debt which is given by

$$V^c(y, b_S, b_L) = \max_{\{b'_S, b'_L, c\}}[u(c) + \beta E_{\{y'|y\}}V^o(y', b'_S, b'_L)] \tag{2.16}$$

Finally,

$$V^o(y, b_S, b_L) = \max\{V^c(y, b_S, b_L), V^d(y)\} \tag{2.17}$$

The default policy of the government can be characterized by repayment sets and default sets. Let $R(b_S, b_L)$ be the set of income levels for which the optimal decision is

repayment when short- and long-term debt are $b_S$ and $b_L$, such that

$$R(b_S, b_L) = \{y \in Y : v^c(y, b_S, b_L) \geq v^d(y)\}$$

and let $D(b_S, b_L) = \tilde{R}(b_S, b_L)$ be the set of income levels for which the optimal decision is default for the debts level of $b_S$, and $b_L$:

$$D(b_S, b_L) = \{y \in Y : v^c(y, b_S, b_L) \leq v^d(y)\}$$

Under competition, a unit bond price satisfies the pricing equations as follows:

$$q_S(y, b_S', b_L') = \frac{1}{1 + r^*} \int_{R(b_S', b_L')} f(y', y) dy', \qquad (2.18)$$

$$q_L(y, b_S', b_L') = \frac{1}{1 + r^*} \int_{R(b_S', b_L')} [\lambda + (1 - \lambda)\{z + q_L(y', \tilde{b}_S', \tilde{b}_L')\}] f(y', y) dy' \qquad (2.19)$$

where,

$$b_S' = \tilde{b}_S(y, b_S, b_L)$$
$$b_L' = \tilde{b}_L(y, b_S, b_L)$$

### 2.4.4   Quantitative Analysis

In this section I describe the functional forms and set all required parameters to calibrate the model. Simulation results follow the calibration of the model.

#### 2.4.4.1   Calibration and Functional Forms

I make the following specifications of functional form and distributional assumptions for the quantitative analysis:

Utility function: $u(c) = c^{1-\gamma}/(1 - \gamma)$

Endowment process: $\ln y_t = \rho \ln y_{t-1} + \epsilon_t$, where $0 < \rho < 1$ and $\epsilon_t \sim N(0, \sigma_\epsilon^2)$

For the numerical specification of the model we need 9 parameter values. There are (i) two endowment process parameters, $\rho$ and $\sigma_\epsilon^2$; (ii) two preference parameters, $\beta$ and $\gamma$; (iii) two parameters describing the bond, the maturity parameter $\lambda$, and the coupon payment $z$; (iv) the probability of reentry following default, $\xi$, and the output cost parameter during the default, $\phi(y)$; and (v) the risk-free rate $r^*$.

The estimated values of the endowment parameters are $\rho = 0.945$ and $\sigma_\epsilon^2 = 0.025$. Using a quadrature based procedure (Tauchen and Hussey, 1991), there is a 51-state Markov chain for the shock. The time preference parameter, $\beta$ is set to $0.953$, and the risk aversion coefficient, $\sigma$ is set to $2$, which is a standard practice in real business cycle studies. The value of $\lambda$ is set to $0.1$ so that average maturity becomes $10$ quarters. However, according to OECD, average sovereign bond maturity has risen to almost $8$ years or $32$ quarters in 2019. Therefore, there is a comparative analysis later with $\lambda = 0.03$. Based on the data on Argentine bonds reported in Broner et al. (2013), the value of $z$ is set to $0.03$. The probability to reenter the financial market after default is set to $0.282$. Following Arellano (2008), the output cost of default, $\phi(y)$ is set to $0.969E(y)$. The risk-free interest rate, $r^*$ is set to $1.0$ percent, corresponding to an annual rate of $4.0$ percent, which is the average nominal yield on three-month US Treasury bills during the period of 1980 to 2001. Table 2.1 summarizes the parameter values.

### 2.4.4.2  Simulation Results

This section analyzes the bond price schedules, value functions, default probabilities, simulation results of output, bond issuance, and the associated prices for previously discussed three methods of bond issuance: only short-term bonds, only long-term bonds, and both short- and long-term bonds.

Table 2.1: Parameters

| Parameter | Description | Value |
|---|---|---|
| $\beta$ | Discount factor | 0.953 |
| $\gamma$ | Risk aversion | 2 |
| $\rho$ | Persistence of output process | 0.945 |
| $\sigma_\epsilon$ | Standard deviation of output shock | 0.025 |
| $\lambda$ | Reciprocal of average maturity | 0.10, 0.03 |
| $z$ | Coupon rate of long-term bonds | 0.03 |
| $\xi$ | Probability of reentry in credit market | 0.282 |
| $\phi(y)$ | Output costs | 0.969 $E(y)$ |
| $r^*$ | Risk-free rate | 0.01 |

Figure 2.2 shows the bond price schedule with respect to the choice of assets, $b'$ (as ratio of mean output) for two different income shocks with 5 percent below and above trend while the sovereign issues either short-term bonds or long-term bonds. It is evident from the figure that there is a sharp decline in bond prices during recessions (low output) for a small amount of increase in borrowings regardless the maturity structure of bonds, and the strict borrowing constraint restricts the sovereign from further borrowing after reaching the borrowing capacity. During economic boom, bond prices remain steady for lower short-term borrowings but decline rapidly for long-term borrowings. However, after a certain level of borrowing, prices decline moderately for long-term bonds but sharply for short-term borrowings. As in Arellano and Ramanarayanan (2012), there is a trade-off between the hedging benefit of long-term debt and the relative incentive benefit of short-term debt. During economic boom because of shock persistence sovereign is expected not to default in a short period of time but may default in the long run and therefore, short-term bonds have more value than long term bonds for small amount of borrowings. Here incentive benefit of short-term debts exceeds the hedging benefit of long-term debts. However, as borrowing increases, the value of short-term debts and its incentive benefit reduce sharply and the value is eventually lower than that of long-term debts at

(a) Only Short-term Bond          (b) Only Long-term Bond
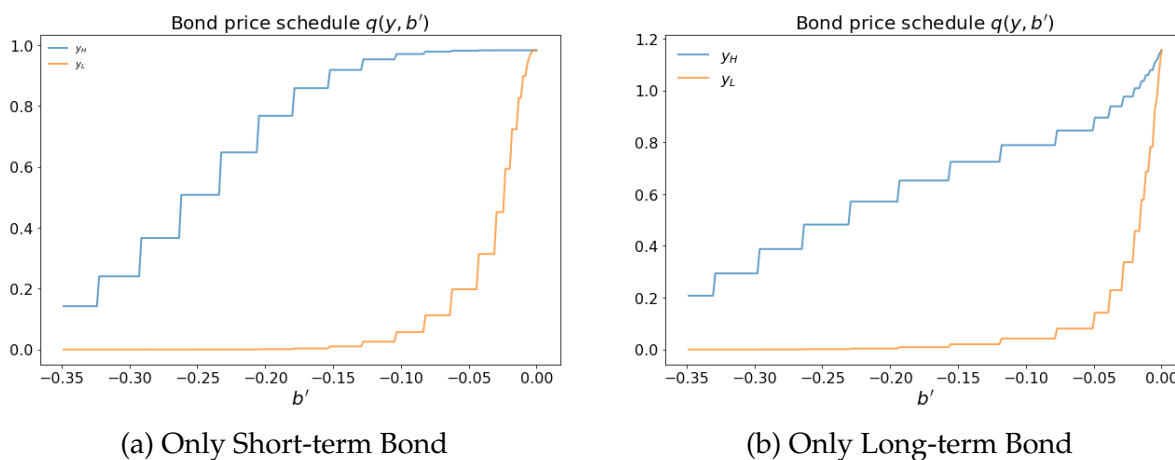
Figure 2.2: Bond Price Schedule with Dilution

extreme borrowing level where hedging benefit of long-term debts exceed the incentive benefit of short-term debts. Therefore, the sovereign can generate more welfare through short-term bonds when the requirement of borrowing is lower and through long-term bonds when it is higher.

Figure 2.3 shows the relative comparison of value functions under two different maturity structure of bonds issuance- short-term bonds and long-term bonds when output is either high or low. During recessions the outcome is the same- after a very small amount of borrowing default becomes an obvious choice in both the cases. However, during boom the sovereign can generate higher welfare with long-term debt since the sovereign can accumulate more borrowings with long-term bonds than with short-term bonds before it needs to go for default.

Figure 2.4 shows the default probability as function of output, $y$ and choice of bonds for next period, $b'$ in heatmap when the sovereign issues short-term bonds or long-term bonds only. The yellow areas are for default and blue areas represent no default combination of output, $y$ and bond level next period, $b'$. The figure shows that default area is higher for short-term bonds than for long-term bonds largely because of the rollover risk

(a) Only Short-term Bond (b) Only Long-term Bond
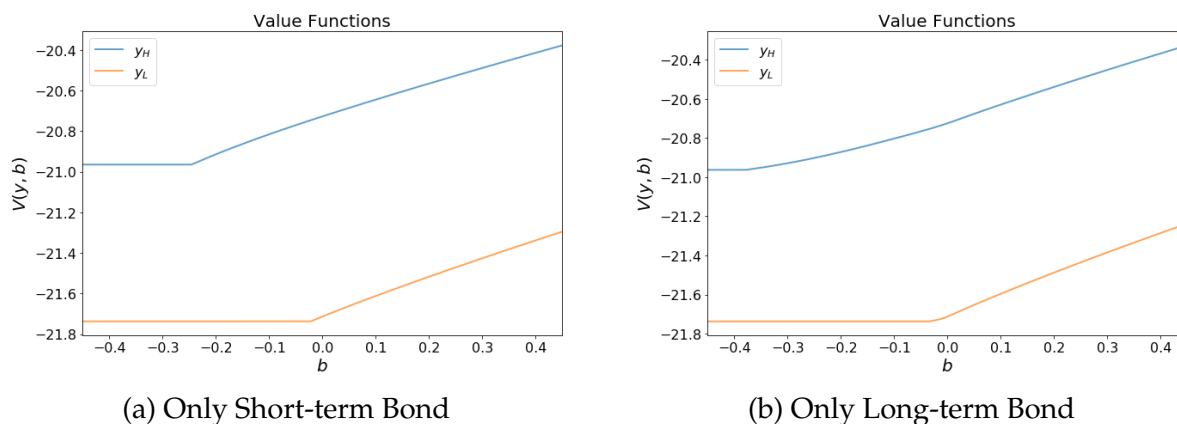
Figure 2.3: Value Functions with Dilution



(a) Only Short-term Bond (b) Only Long-term Bond
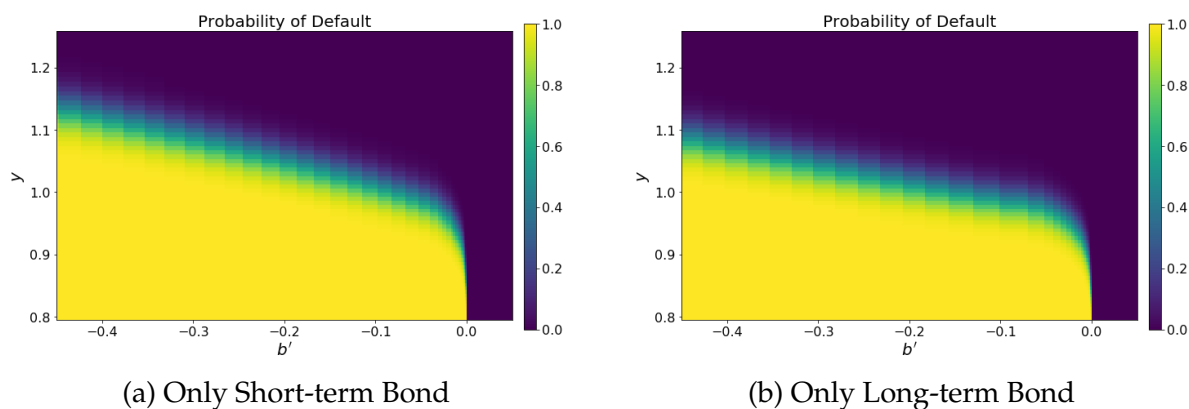
Figure 2.4: Default Probabilities with Dilution

of short-term bonds. In case of short-term bonds, the sovereign needs to roll over the existing bonds by issuing new bonds whereas rollover is not required in case of long-term bonds. Therefore, default probability is higher for short-term bonds.

Figure 2.5 shows time series comparison between issuing either short-term bonds or long-term bonds in terms of output, foreign asset levels and bond price. The gray bars represent the default periods. The simulation results are almost the same for both the cases- when output is high and the economy is in boom, the borrowing capacity and borrowing levels are high with higher bond prices. The opposite case is also true for both

(a) Only Short-term Bond

(b) Only Long-term Bond
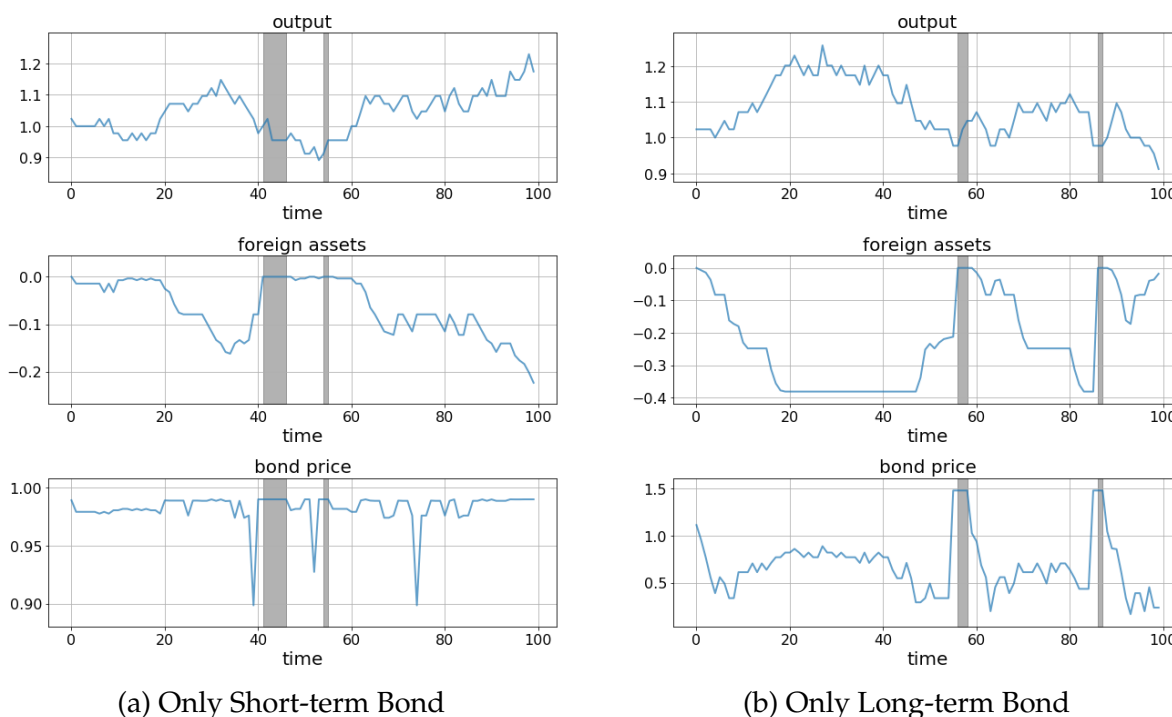
Figure 2.5: Simulation Results with Dilution

the cases- when output is low and the economy is in recession, the borrowing capacity and the borrowing levels are low.

Figure 2.6 shows the short-term bond price schedule when the sovereign issues bonds with both maturity structure- short-term and long term. The price schedule is shown as a function of the level of choice of short-term bonds, $b'_S$ by the sovereign for the next period, given levels of output, $y$ and the choice of long-term bonds, $b'_L$. The left panel shows the price schedule when output is high but long-term bond level is low and when output is low but level of long-term bond is high. During a recession the sovereign chooses to default after the price goes to zero after a small amount of bonds issuance. This is also because the sovereign has high level of long-term bonds already and further borrowing capacity is low. During economic boom, the price is close to one until debt-to-output ratio reaches 15 percent after which the price declines rapidly. The right panel shows the price
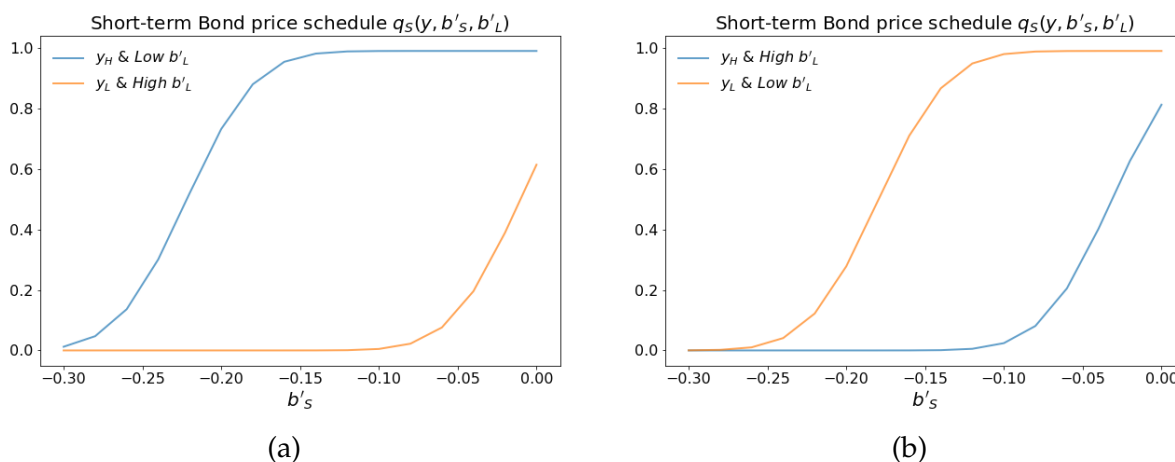
Figure 2.6: Short-term Bond Price Schedule with Dilution

schedule when both output and the level of long-term bonds are either high or low. This is interesting because the price is higher during a recession than during an economic boom due to a lower level of long-term bonds which frees up borrowing capacity with short-term bonds. Even in economic boom the sovereign goes to default with more than 10 percent short-term debt-to-output ratio since the borrowing capacity is low due to higher level of long-term debts.

Figure 2.7 shows the long-term bond price schedule when the sovereign issues bonds with both maturity structure- short-term and long term. The price schedule is shown as a function of the level of choice of long-term bonds, $b'_L$ by the sovereign for the next period, given levels of output, $y$ and the choice of short-term bonds, $b'_S$. The left panel shows the price schedule when output is high or low but the level of short-term bond is always low. The price goes down gradually after a certain level of long-term bonds regardless of state of the economy. However, the price goes down more during recessions which is expected because of lower borrowing capacity and more default risk. The right panel shows the price schedule when both output and the level of short-term bonds are either high or

(a)  (b)
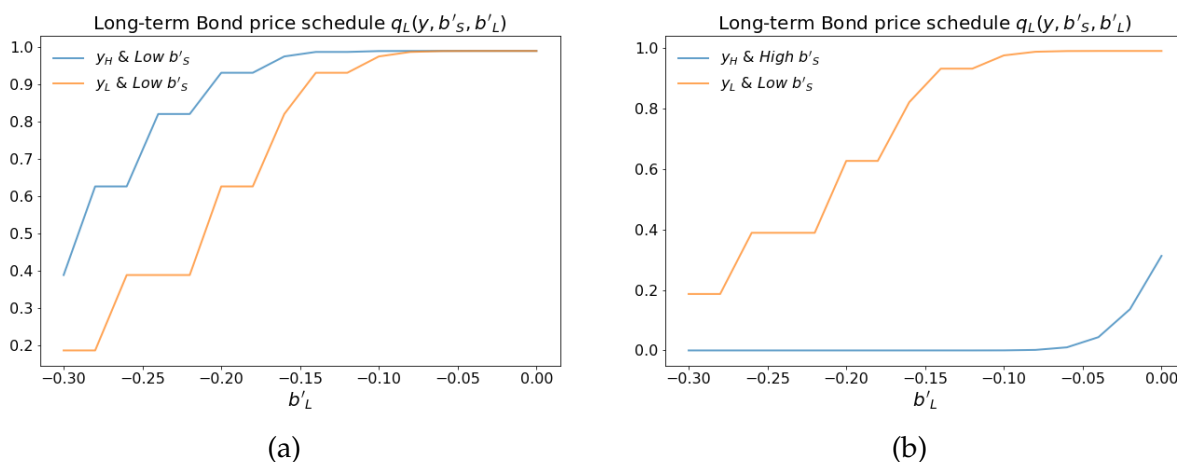
Figure 2.7: Long-term Bond Price Schedule with Dilution



(a) with respect to short-term bond  (b) with respect to long-term bond
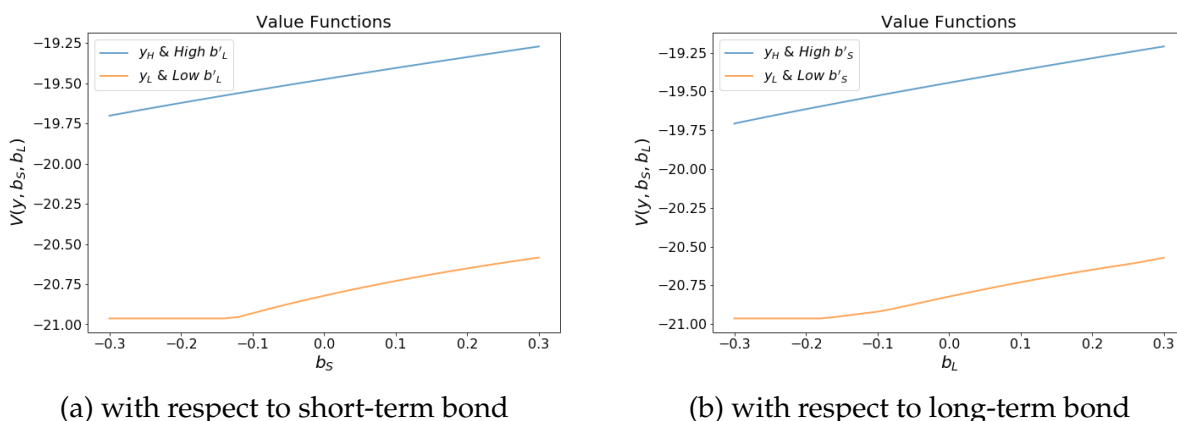
Figure 2.8: Long-term Bond Price Schedule with Dilution

low. Even the economy in boom, with a high level of short-term debt-to-output ratio, the sovereign cannot borrow without default after $5$ percent long-term debt-to-output ratio.

Figure 2.8 shows the value functions with issuance of bonds of both maturity structure-short-term and long-term. The left panel shows the value function in term of short-term bonds when output, $y$ and the level of choice of long-term bonds, $b'_L$ are both high or low. The left panel shows the value function in term of long-term bonds when output, $y$ and the level of choice of short-term bonds, $b'_S$ are both high or low. We do not find any notice-
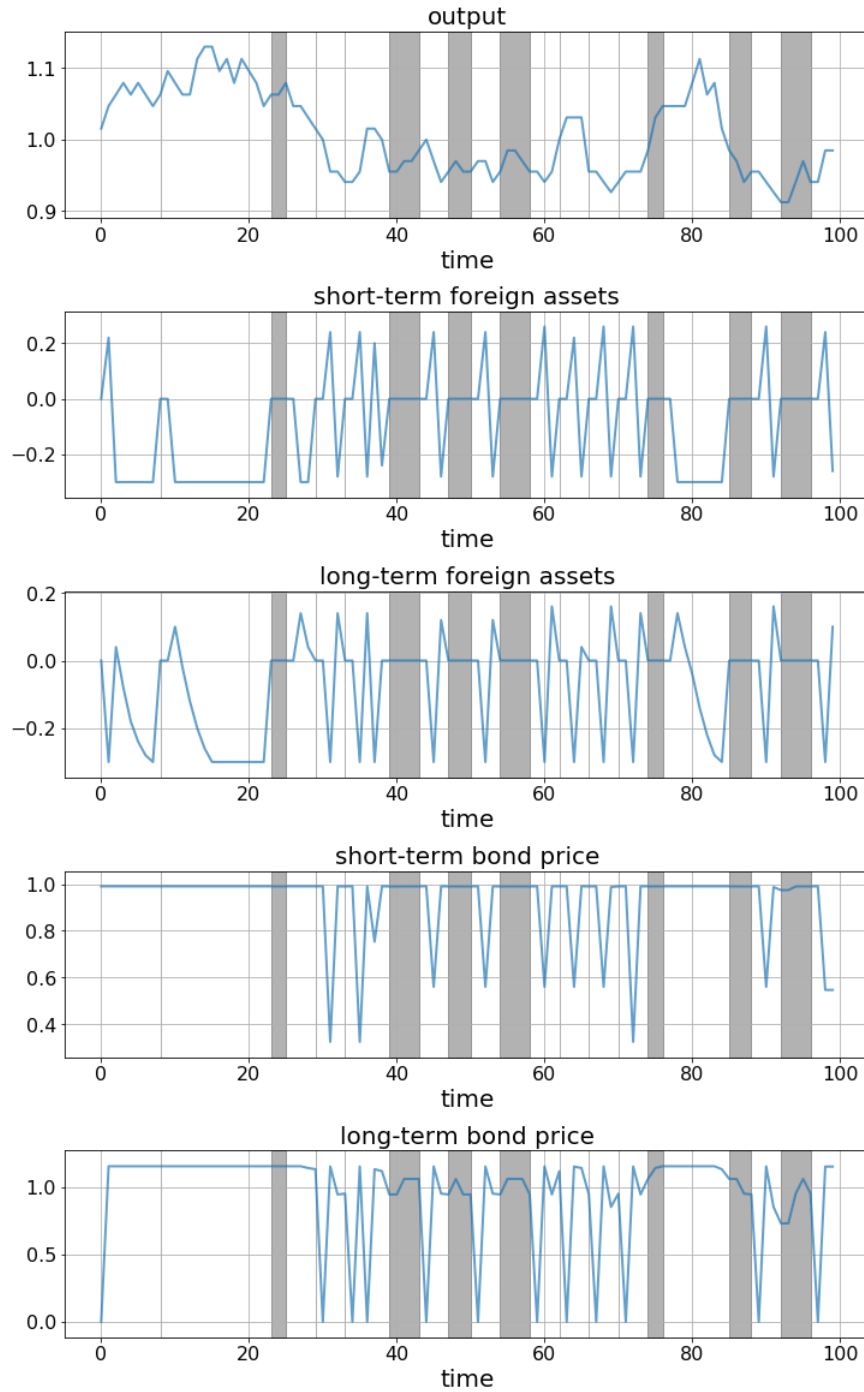
Figure 2.9: Simulation Results with Multiple Maturity Structure and Dilution

able difference in the value functions in terms of the maturity structure of bonds. Hence, value function is trivial in case of deciding the maturity structure of bond issuance.

Figure 2.9 shows the time series simulation of output, holdings of short-term and long-term bonds by the sovereign, and their associated prices. When the output is high and the economy is in boom, the sovereign faces higher borrowing capacity and therefore, holds higher short-term and long-term bonds. The prices are also high because of lower default risk during a boom. During the periods of lower output and economic recessions, the sovereign cannot borrow due to lower borrowing capacity. However, in the periods when output decreases gradually, the sovereign participates in the credit market with opposite position on short-term and long-term bonds, i.e., the sovereign buys short-term bonds while issues long-term bond and vice versa during the period of declining output.

## 2.5   The Models without Dilution

This section compares different methods of eliminating the dilution problem in sovereign debt. The dilution problem is only involved with the long-term debt. However, holders of previously issued long-term debt get worse off regardless the sovereign issues new debt of any maturity structure. To compare among the methods, in this section, we assume that the sovereign issues new debt on either maturity structure, i.e., short- or long-term debt. This will also allow us to focus on the differences of social welfare among the methods.

### 2.5.1   Compensation Covenant with Long-term Bonds

As in Hatchondo et al. (2016), this section proposes a compensation covenant which specifies that if the sovereign would like borrow more, it has to pay the difference between the observed bond price, $q(y, b')$ and the counterfactual bond price, $q(y, (1 - \lambda)b)$ that would have been observed without any new borrowing by the sovereign in the current

period to each holder of previously issued long-term bonds. This covenant makes the long-term bonds value independent of any borrowing in the future by the sovereign and thereby reduces the debt dilution problem.

The payment specified in the covenant is given by

$$\mathcal{C}(y,b,b') = \max\{q(y,(1-\lambda)b) - q(y,b'),0\} \tag{2.20}$$

The government's budget constraint reads as

$$c \leq \begin{cases} y + q(y,b')[(1-\lambda)b - b'] + [\lambda + (1-\lambda)z]b + (1-\lambda)b\,\mathcal{C}(y,b,b') & \text{Repayment} \\ y - \phi(y) & \text{Default} \end{cases} \tag{2.21}$$

Without the dilution problem of sovereign debt through a compensation covenant for the existing bondholders before the sovereign can issue new bonds, there is a new term in the sovereign's budget constraint- $(1-\lambda)b\,\mathcal{C}(y,b,b')$ which implies that the sovereign must compensate the existing bondholders $(1-\lambda)$ portion of bonds with $\mathcal{C}(y,b,b')$. So, the total compensation becomes $(1-\lambda)b\,\mathcal{C}(y,b,b')$.

The price of a bond is given by

$$q(y,b') = \frac{1}{1+r^*}E_{\{y'|y\}}\left[[1-d(y',b')][\lambda + (1-\lambda)\{z + q(y',\tilde{b}') + \mathcal{C}(y',b',\tilde{b}')\}]\right]$$

Price of long-term bonds also adjusts the expected compensation $\mathcal{C}(y',b',\tilde{b}'$ in case of new borrowings by the sovereign in future.

The lifetime utility of a sovereign in a state of default is:

$$V^d(y) = u(y-\phi(y)) + \beta E_{\{y'|y\}}[(1-\xi)V^d(y') + \xi V^o(y',0)] \tag{2.22}$$

The payoff of the sovereign from repaying the debt, denoted $V^c(y,b)$, is given by

$$V^c(y,b) = \max_{\{b',c\}}[u(c) + \beta E_{\{y'|y\}}V^o(y',b')] \tag{2.23}$$

Finally,

$$V^o(y,b) = \max\{V^c(y,b), V^d(y)\} \tag{2.24}$$

(a) Price with Dilution  (b) Price without Dilution
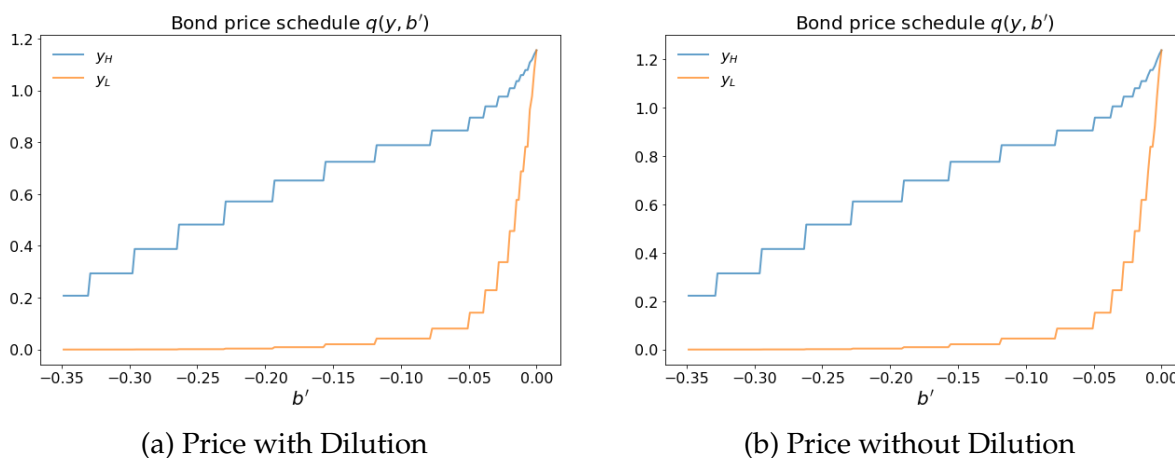
Figure 2.10: Long-term Bond Price Schedule with Dilution

### 2.5.1.1 Results

This section presents results in terms of bond prices, value functions, default probabilities, and simulation of output and foreign assets with the prices when the sovereign issues only long-term bonds with a compensation covenant as a solution to the debt dilution problem. This section also compares the results with the results when the debt dilution problem exists, i.e., without a compensation covenant for the existing bondholders.

Figure 2.10 shows the bond price schedule with dilution and without dilution with respect to the choice of new debt level under two output levels- high and low, when the sovereign issues only long-term bonds. The price schedule is pretty similar in both the cases. During low output period the bond prices go down sharply for a small amount of increase in borrowing and soon go close to zero. During economic boom the prices go down moderately and the difference is insignificant to observe visually. We will see the difference later when we compare the average prices with and without the dilution problem.

31



(a) Value Functions with Dilution

(b) Value Functions without Dilution
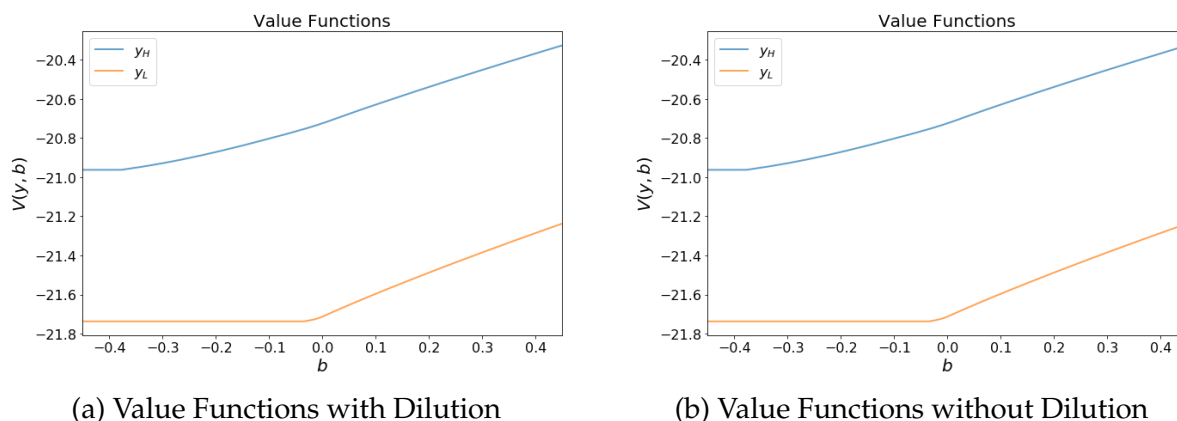
Figure 2.11: Value Functions Comparison



(a) Default Probability with Dilution

(b) Default Probability without Dilution
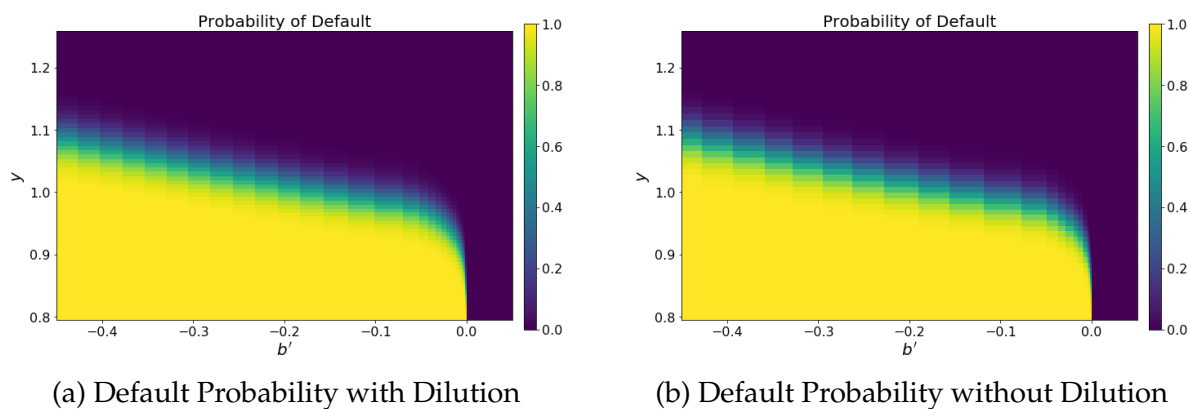
Figure 2.12: Default Probabilities Comparison

Figure 2.11 shows the value functions with dilution and without dilution with respect to the debt level under two output levels- high and low, when the sovereign issues only long-term bonds. The value functions are pretty similar in both the cases. During low output period the value functions go down sharply for a small amount of increase in borrowing and soon become flat due to default decision by the sovereign. During economic boom the value functions go down moderately and the sovereign can borrow more before going for a default decision. The difference is insignificant to observe visually though.

(a) Simulation Results with Dilution

(b) Simulation Results without Dilution
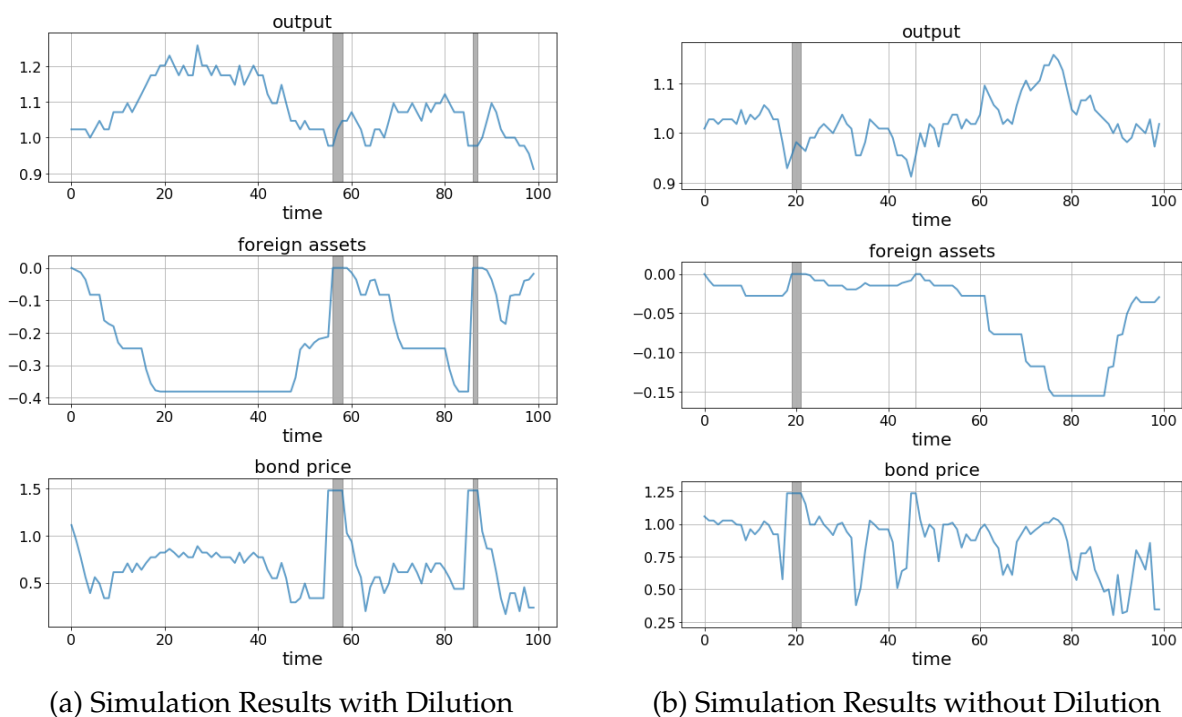
Figure 2.13: Simulation Results Comparison with Long-term Bonds

Figure 2.12 shows the default probabilities with dilution and without dilution with respect to the choice of new debt levels and the levels of output, when the sovereign issues only long-term bonds. The default probabilities are pretty similar visually in both the cases. We will see the difference later when we compare the average default probabilities with and without the dilution problem.

Figure 2.13 shows the simulation results of output, foreign assets, and bond prices with and without dilution, when the sovereign issues only long-term bonds. The results are pretty similar in both the cases. During low output period the bond prices go down and due to lower borrowing capacity borrowing also goes down. During economic boom the prices go up along with the borrowing due to higher borrowing capacity.

33

## 2.5.2 Compensation Covenant with Multiple Maturity Structure

In this section I assume that the sovereign can issue both short- and long-term bonds with a compensation covenant in the contract of long-term bonds. The compensation now depends on current period starting and next period levels of short- and long-term debts besides the stochastic income $y$. Others are as before.

The payment specified in the covenant is given by

$$\mathcal{C}(y, b_S, b_L, b'_S, b'_L) = \max\{q(y, b_S, (1-\lambda)b_L) - q(y, b'_S, b'_L), 0\} \tag{2.25}$$

Government resource constraint is as follows:

$$c \leq \begin{cases} y - q_S(\cdot)b'_S + q_L(\cdot)[(1-\lambda)b_L - b'_L] + b_S + [\lambda + (1-\lambda)z]b_L + \Gamma\mathcal{C}(\cdot) & \text{Repayment} \\ y - \phi(y) & \text{Default} \end{cases} \tag{2.26}$$

where,

$$q_S(\cdot) = q_S(y, b'_S, b'_L)$$
$$q_L(\cdot) = q_L(y, b'_S, b'_L), \quad and$$
$$\Gamma\mathcal{C}(\cdot) = (1-\lambda)b_L\mathcal{C}(y, b_S, b_L, b'_S, b'_L)$$

The lifetime sovereign utility in default is given by:

$$V^d(y) = u(y - \phi(y)) + \beta E_{\{y'|y\}}[(1-\xi)V^d(y') + \xi V^o(y', 0, 0)] \tag{2.27}$$

The payoff of the sovereign from repaying the debt, denoted $V^c(y, b_S, b_L)$, is given by

$$V^c(y, b_S, b_L) = \max_{\{b'_S, b'_L, c\}} [u(c) + \beta E_{\{y'|y\}}V^o(y', b'_S, b'_L)] \tag{2.28}$$

Finally,

$$V^o(y, b_S, b_L) = \max\{V^c(y, b_S, b_L), V^d(y)\} \tag{2.29}$$

The default policy of the government can be characterized by repayment sets and default sets. Let $R(b_S, b_L)$ be the set of income levels for which the optimal decision is repayment when short- and long-term debt are $b_S$ and $b_L$, such that

$$R(b_S, b_L) = \{y \in Y : v^c(y, b_S, b_L) \geq v^d(y)\}$$

and let $D(b_S, b_L) = \tilde{R}(b_S, b_L)$ be the set of income levels for which the optimal decision is default for the debts level of $b_S$, and $b_L$:

$$D(b_S, b_L) = \{y \in Y : v^c(y, b_S, b_L) \leq v^d(y)\}$$

Under competition, a unit bond price satisfies the pricing equations as follows:

$$q_S(y, b'_S, b'_L) = \frac{1}{1 + r^*} \int_{R(b'_S, b'_L)} f(y', y) dy',$$

(2.30)

$$q_L(y, b'_S, b'_L) = \frac{1}{1 + r^*} \int_{R(b'_S, b'_L)} [\lambda + (1 - \lambda)\{z + q_L(y', \tilde{b}'_S, \tilde{b}'_L) + \mathcal{C}(y', b'_S, b'_L, \tilde{b}'_S, \tilde{b}'_L)\}] f(y', y) dy'$$

(2.31)

where,

$$b'_S = \tilde{b}_S(y, b_S, b_L)$$
$$b'_L = \tilde{b}_L(y, b_S, b_L)$$

### 2.5.2.1   Results

This section presents results in terms of bond prices, value functions, default probabilities, and simulation of output and foreign assets with the prices when the sovereign issues only both short- and long-term bonds with a compensation covenant as a remedy to the debt dilution problem.

Figure 2.14 shows the short-term bond price schedule when the sovereign issues bonds with both maturity structure- short-term and long term with a compensation covenant as

Figure 2.15: Long-term Bond Price Schedule without Dilution

The price schedule is shown as a function of the level of choice of long-term bonds, $b'_L$ by the sovereign for the next period, given levels of output, $y$ and the choice of short-term bonds, $b'_S$. The left panel shows the price schedule when output is high or low but the level of short-term bond is always low. The price goes down gradually after a certain level of long-term bonds during a recession which is expected because of lower borrowing capacity and more default risk. The prices are very close to one for $20$ percent long-term debt-to-output ratio and go down afterwards. The right panel shows the price schedule when output is high but the level of short-term debt is low and when output is low but the level of short-term debt is high. During economic recessions when output is low, the sovereign does not have any borrowing capacity due to high short-term borrowing. During economic boom when output is high, the sovereign can issue significant amount of long-term bonds without a major price discount if the level of issuance of short-term bonds is low.

Figure 2.16 shows the value functions with respect to the debt level under two output levels- high and low, when the sovereign issues both short- and long-term bonds with a compensation covenant to mitigate the debt dilution problem. The left panel shows the

(a) With respect to short-term bond    (b) With respect to long-term bond
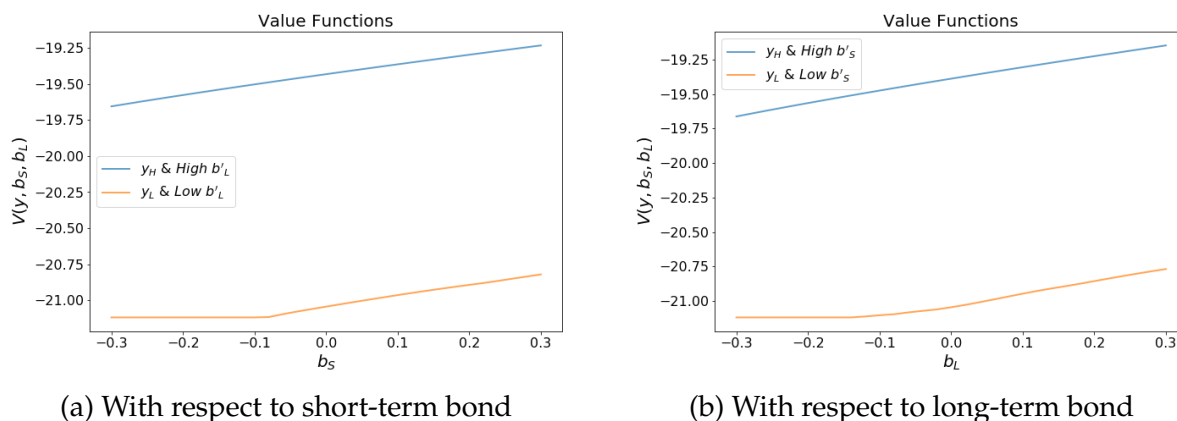
Figure 2.16: Value Functions without Dilution

value functions with respect to short-term bonds and the right panel shows the value functions with respect to long-term bonds. The value functions are pretty similar in both the cases. During low output period the value functions go down sharply for a small amount of increase in borrowing and soon become flat due to default decision by the sovereign. During economic boom the value functions go down moderately and the sovereign can borrow more. The difference is insignificant to observe visually though.

Figure 2.17 shows the time series simulation of output, holdings of short-term and long-term bonds by the sovereign, and their associated prices when bond issuance comes with a compensation covenant. When the output is high and the economy is in boom, the sovereign faces higher borrowing capacity and therefore, holds higher short-term and long-term bonds. The prices are also high because of lower default risk during a boom. During the periods of lower output and economic recessions, the sovereign cannot borrow due to lower borrowing capacity. However, in the periods when output decreases gradually, the sovereign participates in the credit market with opposite position on short-term and long-term bonds, i.e., the sovereign buys short-term bonds while issues long-term bond and vice versa during the period of declining output.
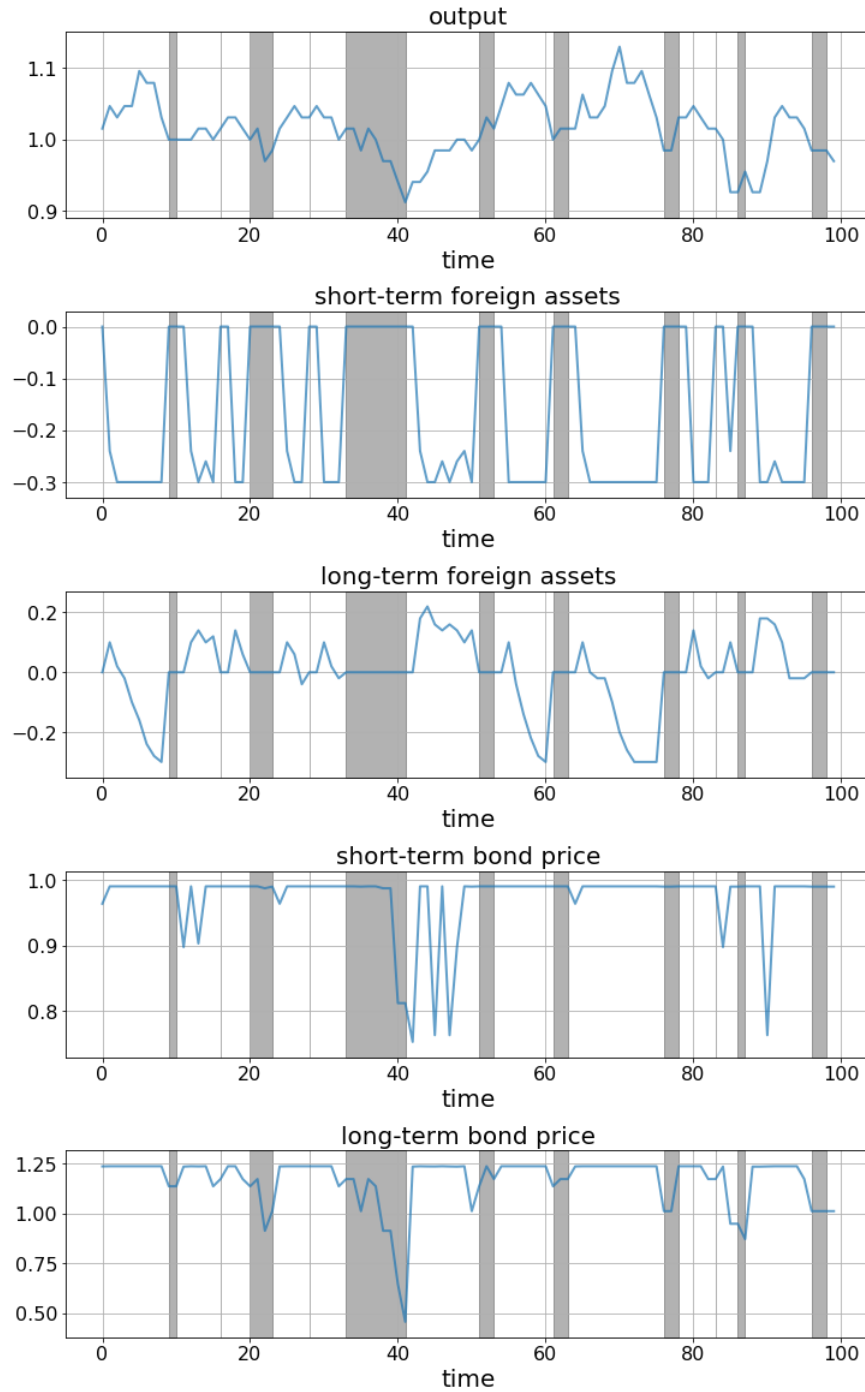
Figure 2.17: Simulation Results without Dilution for Multiple Maturity Structure

## 2.6   Comparative Analysis

Table 2.2 compares the models discussed in terms of the averages of default proba-
bility, short-term price, and long-term price when the value of $\lambda$ is equal to $0.10$. From
table 2.2 it is evident that average default probability is the highest and therefore, aver-
age bond price is the lowest with only short-term bonds. Default probabilities go down
when the sovereign has a probabilistic maturity structure for the bonds. In this type of
bonds, average prices go up if the sovereign decides to include a compensation covenant
in the bond contract that requires sovereign to compensate the existing bondholders be-
fore the government can issue new bonds. The default probability further goes down if
the sovereign issues both short- and long-term bonds. There are also significant increases
in the bond prices as well. If the sovereign has a compensation covenant while issuing
bonds with both the maturity structures then price for long-term bonds goes up while,
interestingly enough, short-term bonds experience a decline in the price. This is largely
because short-term bondholders do not get any compensation if the government decides
to issue new bonds. Short-term bondholders either get the face value of the bond back
in case of repayment or get nothing in case of default by the sovereign at the maturity of
the bond. Moreover, default probability goes up with the inclusion of the compensation
covenant in the bond contract and short-term bond price is largely related to the default
probability of the sovereign.

Figure 2.18 shows the effect of compensation covenant on prices of long-term bonds on
the left panel and short-term bonds on the right panel. Due to the compensation covenant
the lenders would like to increase the supply of long-term bonds while decrease the sup-
ply of short-term bonds. From the result of long-term bond price with compensation

Table 2.2: Comparative Results ($\lambda = 0.10$)

| Model | Avg. $\delta$ | Avg. $q_S$ | Avg. $q_L$ |
|---|---|---|---|
| Only Short-term Bond | 0.299 | 0.694 | - |
| Only Long-term Bond | 0.268 | - | 0.758 |
| Only Long-term Bond & Compensation | 0.268 | - | 0.812 |
| Bonds with Multiple Maturity | 0.193 | 0.799 | 0.859 |
| Bonds with Multiple Maturity & Compensation | 0.206 | 0.786 | 0.910 |
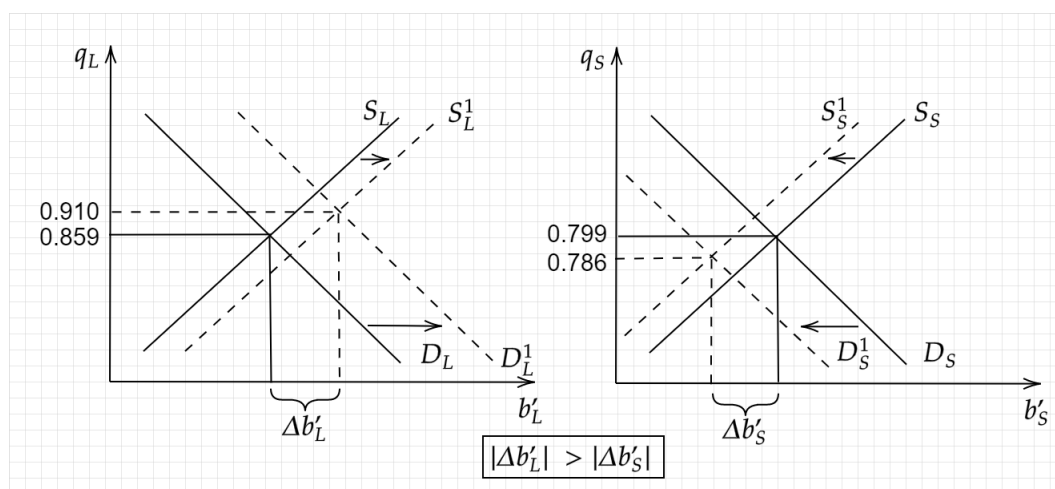


Figure 2.18: Effects of Compensation Covenant on Prices

covenant we can infer that the demand for long-term bonds also goes up. This makes the overall price increase for long-term bonds. The demand for short-term bonds goes down resulting in the lower price of short-term bonds. Finally, since the default probability depends on the levels of bonds, the increase in long-term bonds is higher than the decrease in short-term bonds which is consistent to the higher default probability with compensation covenant when $\lambda = 0.10$.

According to OECD Sovereign Borrowing Outlook 2019, the average sovereign debt maturity is $8$ years. Therefore, we need to increase the maturity of long-term bonds in

Table 2.3: Comparative Results ($\lambda = 0.03$)

| Model | Avg. $\delta$ | Avg. $q_S$ | Avg. $q_L$ |
|---|---|---|---|
| Only Short-term Bond | 0.299 | 0.694 | - |
| Only Long-term Bond | 0.243 | - | 0.919 |
| Only Long-term Bond & Compensation | 0.245 | - | 1.067 |
| Bonds with Multiple Maturity | 0.210 | 0.782 | 0.982 |
| Bonds with Multiple Maturity & Compensation | 0.173 | 0.819 | 1.233 |

out model to compare the results with those of lower maturity debts ($\lambda = 0.10$). Table 2.3 compares the models discussed in terms of the averages of default probability, short-term price, and long-term price when the value of $\lambda$ is equal to $0.03$. From table 2.3 it is evident that default probabilities go down when the sovereign has a probabilistic maturity structure for the bonds. Comparing with table 2.2, it is also evident that the default probability goes down to $24.3$ percent from $26.8$ percent with an increase in the average maturity of long-term bonds from $10$ periods to $33$ periods. Long-term prices also improve in case of both with and without compensation covenant with an increase in the average maturity of bonds. In fact, when the sovereign issues both the short- and long-term bonds, the average price of long-term bonds goes over one, allowing the sovereign to issue premium bonds. Two other interesting findings are the improvement of both the average default probability and the average short-term bond price after the inclusion of the compensation covenant when the average maturity of long-term bonds is high ($\lambda = 0.03$). When the average maturity of long-term bonds is low ($\lambda = 0.10$), the average default probability goes up from $19.3$ percent to $20.6$ percent after inclusion of the compensation covenant. However, with a sufficiently large maturity ($\lambda = 0.03$), the default probability goes down to $17.3$ percent from $21$ percent after including the compensation covenant when the sovereign issues both short- and long-term bonds. With the increase of the maturity of long-term bonds, the average price of short-term bonds goes down to $0.782$

from 0.799 without the compensation covenant in the model of bonds with both maturity structure. However, with inclusion of the compensation covenant, the average short-term price, in fact, goes up from 0.786 to 0.819. This confirms that the compensation covenant solves the debt dilution problem in a larger scale if the maturity of long-term bonds is sufficiently high.

## 2.7    Concluding Remarks

Maturity structure plays an important role in sovereign debt. Moreover, a sovereign's lack of commitment to the action of future governments regarding future borrowing and default give birth to the debt-dilution problem and eventually long-term bonds become costly. In the calibrated model I show that short-term bonds do not face the debt dilution problem since the sovereign issues such bonds only once in a period. However, rollover crisis is higher in case of short-term bonds. I also show that default frequency and debt-dilution problem reduce significantly when the sovereign issues both short- and long-term bonds.

Debt dilution problem has several remedies, such as, compensation package for the existing bondholders (Hatchondo et al., 2016) or a seniority arrangement for the bond-holders (Chatterjee and Eyigungor, 2015). However, none of these solutions are comprehensive to protect the bondholders completely from higher default probability of the sovereign due to raising the debt level. With the compensation covenant in multiple maturity structure in debt issuance, the existing creditors will be compensated before any new issuance of bonds by the sovereign. This mechanism has twofold benefits. First, it provides the existing creditors a compensation during further borrowing by the sovereign. Second, it reduces the default risk when the maturity of long-term bonds is sufficiently high.

Future direction of research would be improving these covenants by specifying the exact compensation packages and bargaining protocols for the bondholders. Endogenizing the maturity of long-term bonds would be another possible interesting work. Moreover, it would be worthwhile to see these cases under both Eaton and Gersovitz (1981) baseline framework and Cole and Kehoe (2000) self-fulfilling rollover crises.

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44

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## 2.9   Appendix

### 2.9.1   The Model with Debt Threshold

A debt threshold covenant requires the sovereign to compensate the holders of each previously issued long-term bonds the difference between the new debt level and a debt threshold before issuing any new debt. This covenant penalizes the sovereign for new borrowing and thus induces a lower debt level. This covenant eliminates the debt dilution problem because the creditors are assured at the beginning of the contract that before the sovereign can borrow from the international credit market, they will get their compensation equal to the new debt level after new borrowings and a certain debt threshold, $\bar{b}$. Thus, the the new compensation covenant is given by

$$\mathcal{C}_b(b') = \psi \max\{\bar{b} - b', 0\} \tag{2.32}$$

Except this debt threshold compensation, everything is the same as in the section where there is a discussion of the model without debt dilution.

### 2.9.2   The Model with Price Threshold

In price threshold covenant the creditors do not need to depend on the counterfactual price of bonds that would be observed without any new debt issuance in a period. The covenant, rather, specifies a compensation for the creditors for a decline in the price of bonds from a certain threshold. This price threshold requires the sovereign to pay the holders of previously issued long-term bonds the difference between a constant threshold price, $\bar{q}$ and the market price of long-term bonds after new issuance, $q(y, b')$. Thus, the promised compensation in the price threshold covenants of long-term bonds is

$$\mathcal{C}_q(y, b, b') = \begin{cases} \max\{\bar{q} - q(y, b'), 0\} & \text{if } b' < (1 - \lambda)b \\ 0 & \text{Otherwise} \end{cases} \tag{2.33}$$

### 2.9.3 The Model with Income Contingent Debt

The government's budget constraint reads as

$$c \leq \begin{cases} y + [\lambda + (1-\lambda)z]b + E_{\{y'|y\}}\{q(y, b'(y'))[(1-\lambda)b - b'(y')]\} & \text{Repayment} \\ y - \phi(y) & \text{Default} \end{cases} \tag{2.34}$$

where,

$$b'(y') \leq \bar{b}'(y') = \sup\{\tilde{b}' : V^c(y', \tilde{b}') \geq V^d(y') \ \forall y\}$$

The lifetime sovereign utility in default is given by:

$$V^d(y) = u(y - \phi(y)) + \beta E_{\{y'|y\}}[(1-\xi)V^d(y') + \xi V^o(y', 0)] \tag{2.35}$$

The payoff of the sovereign from repaying the debt, denoted $V^c(y, b)$, is given by

$$V^c(y, b) = \max_{\{b'(y'), c\}} [u(c) + \beta E_{\{y'|y\}} V^o(y', b'(y'))] \tag{2.36}$$

Finally,

$$V^o(y, b) = \max\{V^c(y, b), V^d(y)\} \tag{2.37}$$

### 2.9.4 The Sovereign Debt with Settlement

Government resource constraint will still hold.

$$c \leq \begin{cases} y - q_S(\cdot)b'_S + q_L(\cdot)[(1-\lambda)b_L - b'_L] + b_S + [\lambda + (1-\lambda)z]b_L & \text{Repayment} \\ y - \phi(y) & \text{Default} \end{cases} \tag{2.38}$$

where, $q_S(\cdot) = q_S(y, b'_S, b'_L)$ and $q_L(\cdot) = q_L(y, b'_S, b'_L)$.

The sovereign's lifetime utility in default is given by:

$$V^d(y) = u(y - \phi(y)) + \beta E_{\{y'|y\}}[(1-\xi)V^d(y') + \xi V^o(y', G_S(y'), G_L(y'))] \tag{2.39}$$

The payoff of the sovereign from repaying the debt, $V^c(y, b_S, b_L)$, and finally, $V^o(y, b_S, b_L)$ will still be the same. The payoff of the sovereign from repaying the debt, denoted $V^c(y, b_S, b_L)$, is given by

$$V^c(y, b_S, b_L) = \max_{\{b'_S, b'_L, c\}} [u(c) + \beta E_{\{y'|y\}} V^o(y', b'_S, b'_L)] \tag{2.40}$$

Finally,

$$V^o(y, b_S, b_L) = \max\{V^c(y, b_S, b_L), V^d(y)\} \tag{2.41}$$

With some abuse of notation, the default decision rule of the sovereign, $d(y, b_S, b_L)$ is implicitly determined by this equation, where $d = 1$ means default is the optimal decision and 0 otherwise.

Under competition, the pricing equations of a unit bond are as follows:

$$q_S(y, b'_S, b'_L) = \frac{1}{1 + r^*} E_{\{y'|y\}} \left[ [1 - d(y', b'_S, b'_L)] + d(y', b'_S, b'_L) \frac{P_S(y', G_S(y'), G_L(y'))}{-b'_S} \right] \tag{2.42}$$

$$q_L(y, b'_S, b'_L) = \frac{1}{1 + r^*} E_{\{y'|y\}} \left[ [1 - d(y', b'_S, b'_L)][\lambda + (1 - \lambda)\{z + q_L(y', \tilde{b}'_S, \tilde{b}'_L)\}] \right. \tag{2.43}$$
$$\left. + d(y', b'_S, b'_L) \frac{P_L(y', G_S(y'), G_L(y'))}{-b'_L} \right]$$

where, as in previous section,

$$b'_S = \tilde{b}_S(y, b_S, b_L)$$
$$b'_L = \tilde{b}_L(y, b_S, b_L)$$

and, $P_S(y, G_S(y), G_L(y))$ and $P_L(y, G_S(y), G_L(y))$ are the aggregate values of expected repayments on the defaulted short- and long-term debts conditional on output being $y$ and, settlement values are $G_S(y)$ and $G_L(y)$, respectively for short- and long-term debts. Since the aggregate values are equally distributed across all bonds in each maturity class, in

expectation, each short-term bond will receive $E_{\{y'|y\}}P_S(y', G_S(y'), G_L(y'))/b'_S$ and each long-term bond will receive $E_{\{y'|y\}}P_L(y', G_S(y'), G_L(y'))/b'_L$.

$P_S(y, G_S(y), G_L(y))$ and $P_L(y, G_S(y), G_L(y))$ are given by:

$$
\begin{aligned}
P_S(y, G_S(y), G_L(y)) = \frac{1}{1+r^*}E_{\{y'|y\}}\Big[(1-\xi)P_S(y', G_S(y'), G_L(y')) \quad (2.44)\\
+\xi q_S(y', G_S(y'), G_L(y'))(-G_S(y'))\Big]
\end{aligned}
$$

$$
\begin{aligned}
P_L(y, G_S(y), G_L(y)) = \frac{1}{1+r^*}E_{\{y'|y\}}\Big[(1-\xi)P_L(y', G_S(y'), G_L(y')) \quad (2.45)\\
+\xi q_L(y', G_S(y'), G_L(y'))(-G_L(y'))\Big]
\end{aligned}
$$

In case, settlement is reached between the sovereign and the creditors next period, then the creditors as groups receive settlement with aggregate values of $q_S(y', G_S(y'), G_L(y'))$ $(-G_S(y'))$ for short-term bonds and $q_L(y', G_S(y'), G_L(y'))(-G_L(y'))$ for long-term bonds.

للاستشارات

# CHAPTER 3.  INTERBANK LIQUIDITY CRISIS AND INTERNATIONAL CONTAGION

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## 3.1  Abstract

Financially integrated economies observe a cross-country credit boom prior to financial recessions and a bust afterwards. This chapter presents a two-country real business cycle model with banking sector where privately known intermediation efficiency of banks make them heterogeneous and gives rise to an interbank market. Overaccumulation of assets or low productivity in one country may lead to credit freeze in both financially integrated countries due to the existence of moral hazard and asymmetric information in the interbank market. A "sail together" financial integration may go into a "sink together" interbank credit freeze.

**JEL Classification:** E44, F34, F41, G01, G15, G21

**Keywords:** Financial integration, interbank market, asset accumulation, productivity, moral hazard, asymmetric information, credit freeze.

## 3.2 Introduction

Recent macroeconomic literature has emphasized on the propagation and amplification of random adverse financial shocks which primarily causes the banking crises (Gertler and Kiyotaki, 2010, 2015). However, recent studies on banking crises has highlighted a close linkage between such crises and credit conditions (Gorton, 2010, 2012). Recent empirical research also corroborate the existence of typical patterns across diverse episodes. There are closed economy models of banking crises (Gertler and Kiyotaki, 2015; Boissay et al., 2016). However, in a financially integrated world, banking crises may propagate and affect other countries. Banking crises are rare events, they follow credit-intensive booms, and involve with multiple nations occasionally. These stylized facts therefore call for an alternative approach of banking crises.

When countries are financially integrated, the interbank market equilibrium may be different from that of a closed economy. Moreover, the interbank crises may result from the states of other countries and therefore the threshold level of the states may also vary for crises. A financially integrated world may lead to spillover effects of banking crises from one country to another. These ideas lead to the research questions: how does the interbank market operate in a financially integrated two-country model? When do countries face inter-banking crises in an integrated world? and is there any spillover effect of inter-banking crises from one country to another? In this paper, we seek for the answers of these questions under a two-country model.

In this chapter , when two countries are integrated through interbank lending, banking crises result from the procyclicality of bank balance sheets of either/both of the countries and may have spillover effects of interbank market freeze on each other. Countries get financially integrated to "sail together" and survive during recessions. However, due to overaccumulaion by one country both the countries may fall into an interbank freeze to

"sink together". During expansions, credit supply increases pushing both the corporate and interbank lending rates down. With lower rates the agency problem in the interbank market gets aggravated leading to contractions in interbank funding. Financial integration may lead to a disastrous interbank market freeze if total credit boom is larger than the productive use of loans.

In our model, we follow the real business cycle setup with heterogeneous banks as in Boissay et al. (2016). However, we have a two-country world where the countries are symmetric. Banks are heterogeneous due to the privately observed intermediation efficiency. They can obtain funds from the depositor/shareholders and/or the interbank market and lend to homogeneous good producing firms. Due to the usual agency problem in the interbank market, banks can borrow and divert funds to lower return assets and such diversion cannot either traced or recovered by the lending banks. Less efficient banks have more incentive to divert, and this diversion depends on the rate of returns in the economy. To restrict such diversion, interbank market has a borrowing capacity resulting from the incentive compatibility constraint. However, lower return in the corporate loans market increases the incentive for diversion and therefore market responds through lowering the borrowing capacity and eventually the market goes into a freeze. In a financially integrated world freeze in one country leads to freeze in another country for lower rates of returns.

The mechanism that leads to a banking crises in a two-country world is as follows. A sequence of favorable, productivity shocks lead to an expansion of credit. The more efficient banks borrow from the less efficient ones to expand their corporate loan operations. The size of the overall banking sector increases and the economy booms. However, when the aggregate productivity growth goes down to converge to its average course, the demand for corporate loans and the corresponding rate goes down. As a result, the interbank lending rate goes down resulting into more incentive for less efficient banks

to borrow and divert funds. Therefore, counterparty risk goes up since the identity of these diverting banks are unknown; interbank lending declines and the market goes into a freeze. When two countries are integrated through interbank lending, a market freeze in one country leads to a freeze in another country as well. Either two countries have an operating interbank market or both goes into a freeze. Excessive credit creation by one country triggers a two-country wide credit freeze. The countries may also go into a freeze from very low productivity in either or both of the countries.

We calibrate the interbank market equilibrium of the model and assess its quantitative properties. Potentially, the model can generate banking crises in one country due to an endogenous factor- overaccumulation of assets or an exogenous factor- low productivity or a combination of both the endogenous and exogenous factors in a period. Moreover, the banking crises in one country shows a spillover effect on another country, i.e., a country that could have a well operating interbank market without any integration, falls into interbank market freeze due to either of the factors of another country after any financial integration. Thus, as in data, banking crises in one country in our model are closely linked to asset accumulation and productivity of not only its own but also another country to which the country is financially integrated. However, we find that this story largely depends on how the countries are integrated, e.g., when households are allowed to deposit in domestic country only, countries are integrated through only the interbank market, and firms can only borrow capital from the domestic banks.

The chapter proceeds as follows. Section 3.3 depicts some related literature of macroeconomic models with financial frictions, and international spillover effect, contagion. Section 3.4 documents the facts about financial recessions of 14 OECD countries for the period 1870-2008 as in Boissay et al. (2016). Section 3.5 describes the basic model with representative household, representative firm, and heterogeneous banks. Section 3.6 characterizes the interbank market equilibrium for both closed and financially integrated two

countries those are integrated in different dimensions. Section 3.7, presents the quantitative analysis and the results of the interbank market equilibrium to alternative state spaces. Section 3.8 concludes the paper.

## 3.3    Related Literature

This chapter is related to the macroeconomic literature with financial frictions. For instance, Bernanke et al. (1999), Gertler and Karadi (2011), Jermann and Quadrini (2012), and Christiano et al. (2014) show how financial market frictions can amplify the financial shocks and generate long-lasting recessions. However, the models are linearized in this class of research and this paper departs from this approach to characterize the important and critical nonlinearities in the mechanism of the interbank market. The important difference is to show the boom-bust cycle as an outcome of endogenous factors rather than of a big financial shock only.

In this respect, this chapter is related to the literature featuring powerful amplification mechanism[1] resulting from financial frictions, e.g., He and Krishnamurthy (2012), and Brunnermeier and Sannikov (2014). This chapter is also related to the literature on sudden stops in emerging market economies. For example, as in Mendoza and Smith (2006) and Mendoza (2010), we find that a standard (mild) productivity shock can trigger a crisis if the agent is highly leveraged. These are also open economy models similar to our model. However, this chapter departs in that it does not have exogenous interest rate; rather endogenous model driven interest rates which play a central role in the mechanism of the interbank market equilibrium.

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[1]Financial crises are inherently nonlinear events, often featuring sudden plummet in asset prices and sharp collapse in output, while with a slow recovery rate. The most recent approach to capture this nonlinearity is to allow for an occasionally binding constraint.

This chapter is relevant to the literature that study the financial fragility due to pecuniary externalities, such as Bianchi and Mendoza (2010) or Bianchi (2011). These papers focus on creation of excess credit demand due to financial frictions affecting nonfinancial firms. In this chapter we focus on excess supply of credit instead where the household fails to internalize the effect of her saving decisions on both the interbank lending rate and the corporate lending rate.

This chapter is also related to the financial contagion literature for international market. For example, Calvo (1998) and Chang and Velasco (2001), among others, study the interaction of the banking system and currency markets in a crisis. However, we do not introduce any currency market; rather the crisis spreads out from one country to another through the banking channel. Allen and Gale (2000), and Perri and Quadrini (2018) study the financial contagion as an outcome of liquidity preference shock. Perri and Quadrini (2018) show that in a two-country model with financial frictions, a global liquidity shortage induced by pessimistic self-fulfilling expectations can quantitatively generate longer and deeper recessions following credit booms. The main differences between theirs and our work are that they focus on balance sheet effect from the private sector, i.e., the role of balance sheets in constraining borrowers from spending when financial markets are imperfect, while our work highlights the role of the wholesale banking sector at the heart of the recent financial crisis in which credit dried up and market froze.[2]

A recent work[3] of Gertler et al. (2019) incorporates banking panics into a standard New Keynesian model and captures both qualitatively and quantitatively the effects of financial collapse on investment, output, and employment. They show that a credit boom can increase financial instability of the system, as in Boissay et al. (2016). The sudden and

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[2]See Gertler and Kiyotaki (2010), Gertler and Karadi (2011), and Curdia and Woodford (2010)for papers that incorporate banking and Iacoviello (2005), Eggertsson and Krugman (2012), and Guerrieri and Lorenzoni (2017) for papers that included household debt.
[3]See also, Gertler and Kiyotaki (2015) and Gertler et al. (2016).

discrete nature of the financial collapse characterized by the paper is induced without observing large exogenous productivity shocks, either. Instead, banks that hold imperfectly liquid long term assets and issue short term debts facing liquidity mismatch are vulnerable to panic failure of household to roll over deposits. A plausible magnitude of negative capital quality shock reduces the value of banker's capital, which either brings banks immediately into insolvency, or leads to a self-fulling rollover crisis in which the liquidation of assets at firesale prices forces banks into default, similar to the Cole and Kehoe (2000) model of sovereign default but contrast with the "early withdrawal" mechanism in the Diamond and Dybvig (1983) model[4]. One of their paper's contributions to the macro-finance literature is that they show that banking panics (or bank runs) are quantitatively more important than non-linearities coming from occasionally binding constraints, as in Brunnermeier and Sannikov (2014) and He and Krishnamurthy (2012). They also manage to endogenize the probability of bank runs by relating it to macroeconomic fundamentals. However, there are three main weaknesses associated with Gertler et al. (2019). First, it is not clear what the negative capital quality shocks stand for in the model if we are trying to understand the interactions between the financial market and the real economy. Second, the paper tends to replicate what happened during the 2007-08 financial crisis but fails to match the exact timing of when the crisis occurred. This issue may be addressed by incorporating some adjustment cost. Third, their model generates a counterfactual faster recovery rate from the recession than in the data. The lack of persistence may be resolved by assigning an equally important role to the real side of the economy in which household deleverages as well in the crisis.

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[4]The traditional bank run models, starting from Diamond and Dybvig (1983), focus on the retail banking sector in which a "sequential service constraint" takes in effect to generate runs, i.e., only households who get to banks early will have their money back, a first-come, first-serve scenario. However, recent bank run mostly occurs in the wholesale banking (or interbank market) sector, such as the failure of Continental Illinois in 1984 and the collapse of Lehman Brothers in 2008.

## 3.4    Financial Recession Facts

Based on data from Jordà et al. (2011, 2013) and Schularick and Taylor (2012) of 14 OECD countries from 1870 to 2008, there are two facts (first two) regarding the financial recessions. The data for the third fact is taken from Bank for International Settlements (BIS).

### 3.4.1    Fact 1: Financial recessions are rare events

Financial recessions are rare events in comparison to other recessions. From data collected of 196 recessions of 14 OECD countries, average probability of a financial recession is 2.36 percent whereas other recessions have an average probability of 8.93 percent.

### 3.4.2    Fact 2: Financial recessions follow credit booms

Financial recessions do not hit at a random; rather break out during credit boom. Credit is 3.25 percent above the trend in the peak year prior to a financial recession in comparison to only 0.61 percent above the trend in case of other recessions.

### 3.4.3    Fact 3: International financial recessions follow international credit booms

International financial recessions follow an increase of international credit by more than 8 percent from the average in the peak year prior to the recession and during the financial recession the international credit decreases by almost 13 percent from the average. Figure 3.1 shows an international credit boom before the financial crisis of 2008.
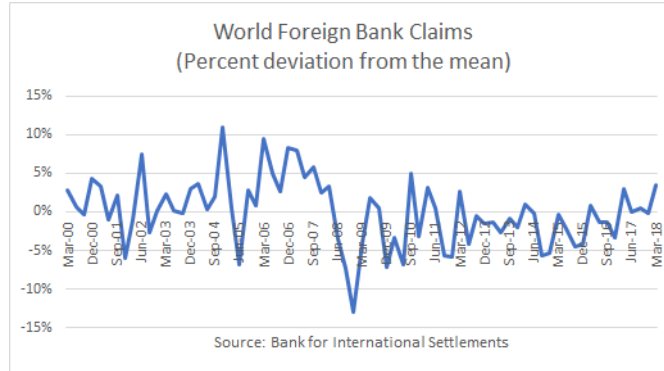
Figure 3.1: International Bank Claims Growth

## 3.5 The Baseline Model

This section explains a symmetric two-country ($j = 1, 2$) Real Business Cycle (RBC) model with banking sector. Each country is populated with one risk averse representative household, one risk-neutral representative competitive firm, and risk-neutral, heterogeneous, and competitive banks with a mass one. For ease of notation, we will exclude any country subscripts for symmetric country characterization; country notations will be introduced when required.

### 3.5.1 The Representative Household

There is an infinitely lived, risk averse, representative household in each of the countries. Household in each country has preferences over consumption, $c$, with an utility function

$$\mathbb{E}_t \sum_{\tau=0}^{\infty} \beta^{\tau} u(c_{t+\tau}) \tag{3.1}$$

where, the usual regularity conditions are satisfied (i.e., $u'(c) > 0$, $u''(c) < 0$, $u'(0) = \infty$, $u'(\infty) = 0$), the psychological discount factor of the household is denoted $\beta \in (0, 1)$ which is not country-specific, and $\mathbb{E}_t(\cdot)$ denotes the expectation operator. The household starts each period $t$ with an individual asset $a$. Aggregate assets are denoted $A$. We can think $a$

as either bank equity or bank deposit and the composition of $a$ is indeterminate due to the lack of friction between the household and the banks. We refer $a$ as the bank deposit and assume that the gross return on deposits is denoted $r$. Following the macro-finance literature (e.g., Gertler and Kiyotaki, 2010; Gertler and Karadi, 2011), we assume that household cannot finance the firms directly due to the frictions between them. We also assume that the supply of labor is inelastically one unit by the household who earns unit wage $w$ for labor, obtains firm profit $\pi$, and receives a lump-sum transfer $\chi$ corresponding to the financial intermediation cost of the banks (more explanations are in the banking sector section). A representative household therefore maximizes her utility (3.1) by deciding on her consumption/saving subject to the budget constraint

$$c + a' = ra + w + \pi + \chi \tag{3.2}$$

### 3.5.2 The Representative Firm

This is a two-country model where the firm lives only for a period and produces a homogeneous good that can be either consumed or invested. To produce the good the firm needs to hire capital $k$ and labor $h$. The production function $zF(k, h)$ follows a constant returns to scale technology and satisfies the standard Inada conditions. The countries may differ due to their country-specific total factor productivity (TFP) shocks, $z$. However, TFP in both countries are assumed to follow the same exogenous AR(1) process:

$$\log z' = \rho_z \log z + \varepsilon' \tag{3.3}$$

where the persistence parameter $|\rho_z| < 1$ and the innovation $\varepsilon$ is normally distributed with mean zero and standard deviation $\sigma_z$. At the beginning of period $t$, $\varepsilon$ is realized, before the firm decides on the requirement of capital $k$ and labor $h$. At the beginning of the period $t$, firm is born without any resources and depends on banks to borrow capital

$k$ at a gross corporate loan rate $R$. Capital $k$ depreciates at rate $\delta \in (0,1)$. At the end of the period, the firm repays the corporate loan. For production, the firm also rents labor from the household and in return pays $w$ per unit of labor. The firm chooses capital, $k$ and labor, $h$ for production to maximize profit,

$$\pi = zF(k,h) + (1-\delta)k - Rk - wh \tag{3.4}$$

### 3.5.3 The Banking Sector

The banking sector plays a very significant role in this model due to two salient features. First, there is heterogeneity in the intermediation skill among banks, i.e., some banks have more intermediation skill than others. This heterogeneity among banks creates an interbank market where the least efficient banks lend to the most efficient ones. Second, following the corporate finance literature (e.g., Tirole, 2006), there is moral hazard problem in the banking sector along with asymmetric information, which essentially impede the interbank market functioning. Overall, banks perform both the retail banking and wholesale banking. On the retail side, they collect deposits from the household and lend to the firms, and on the wholesale side, they issue interbank claims to reallocate assets among themselves where low skilled banks lend to the high skilled ones.

#### 3.5.3.1 Banks

There is a continuum of risk-neutral, competitive banks in both countries. They are born at the end of period $t-1$ and only live for one period, i.e., banks die at the end of period $t$. Each bank, after birth, collects deposit $a$ from the household. They then become heterogeneous by drawing a random, bank-specific, intermediation skill $p$. Henceforth, we denote bank $p$ as bank with intermediation skill $p$. The skill $p$ is distributed over the interval $[0,1]$ with cumulative distribution $\mu(p)$, satisfying $\mu(0) = 0$, $\mu(1) = 1$, and

$\mu'(p) > 0$. If the bank $p$ decides to lend to the firm, then at the end of the period there is an intermediation cost $(1-p)R$ per unit of corporate loan, so that bank $p$ earns an effective gross return of $pR$ from corporate loan.

To avoid any deadweight losses in the economy, household receives a lump-sum rebate $\chi$, which is essentially equal to the intermediation cost. These intermediation costs may arise from the prospection, screening, and monitoring while originating or servicing the loan. The banks can also invest into an outside project with a constant gross return $\gamma$. For fixing the idea, we refer this outside option as the storage technology which is at least as good as just letting the good depreciate, i.e., $\gamma \geq 1 - \delta$.

Due to the heterogeneity of banks in terms of intermediation skill $p$, in each country there is an intraperiodic interbank market, where low $p$ banks lend to the high $p$ ones at gross rate $\rho$. In equilibrium, this interbank loan rate must be lower than the corporate loan rate $R$; otherwise all banks would find it optimal to lend in the interbank market at higher rate $\rho$ than lending to the firms at lower corporate rate $R$. Similarly, the storage return $\gamma$ must be lower than the interbank lending rate $\rho$; otherwise all banks would find it optimal to store and not participate in the interbank lending market. Banks $p$ take the interbank lending rate $\rho$ and the corporate lending rate $R$ as given and decides whether, and how much to borrow or lend.

Let $\Phi$ be the borrowing per deposit by a borrowing bank $p$, where $\Phi \geq 0$ also denotes the publicly observable and endogenous market funding ratio. If a bank $p$ decides to borrow $\Phi$ (per unit of deposit) from other banks at cost $\rho$ then gets a gross unit return on deposits equal to $pR(1 + \Phi) - \rho\Phi$, and lends $1 + \Phi$ (per unit of deposit) to the firm for gross return $pR$. If a bank decides to lend to other banks, instead, then the gross return on deposit is just $\rho$.

Gross return on deposits for bank $p$ is then

$$r(p) \equiv \max \{pR(1 + \Phi) - \rho\Phi, \ \rho\} \tag{3.5}$$

Bank $p$ chooses to be a borrower when

$$pR(1 + \Phi) - \rho\Phi \geq \rho \Leftrightarrow p \geq \bar{p} \equiv \frac{\rho}{R} \tag{3.6}$$

Inequality (3.6) imposes a participation constraint on bank $p$ to be a borrower, not a lender in the interbank market. Banks with $p < \bar{p}$ are less efficient in intermediation and hence lend to the more efficient banks $p \geq \bar{p}$, and the marginal bank $\bar{p}$ is indifferent between the two options. In a frictionless world, $\bar{p} = 1$ since all banks with $p < 1$ would lend to the most efficient one with $p = 1$ in that case. The economies would have reached the first-best allocations then. There are two frictions prevalent on the interbank market which prevent the economies from reaching their first-best allocation: moral hazard and asymmetric information.

Before explaining the details and solutions of the moral hazard problem and asymmetric information in the interbank market we need to see the overall market flow in figure 3.2. Households do not know the skill level of banks $p$, while this skill level is unknown to other banks as well. Therefore asymmetric information rises. Banks with $p \geq \bar{p}$ borrows $\Phi$ per deposit from the interbank market, lends $(1 + \Phi)$ per deposit to the firms, get $(1 + \Phi)R$ from the firms in return, and finally returns $\rho\Phi$ to the lender. However, banks $p < \bar{p}$ may pretend to be high-skill bank and borrow from the interbank market and invest in an alternative storage technology at a return $\gamma$. This gives rise to the moral hazard in the interbank market.

### 3.5.3.2 Moral Hazard

We assume that the creditors cannot trace the proceeds of the storage technology and therefore cannot seize those. As a consequence, borrowing banks may choose to renege
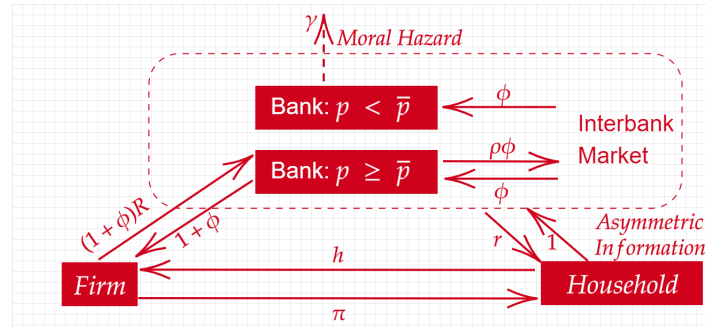
Figure 3.2: Market Flows

on their interbank debt contracts and walk away from the lenders. So, interbank loan contracts are not enforceable. A bank $p$ that walks away with $(1 + \Phi)a$ and invests in the storage technology, gets $\gamma(1 + \theta\Phi)a$ as payoff, where $\theta \in [0, 1]$ is the cost for walking away from the interbank market after borrowing (the higher $\theta$, the more this cost). Such opportunistic behavior is referred as "diversion" (e.g., Hart, 1995; Burkart and Ellingsen, 2004).

Corporate finance literature (e.g., Tirole, 2006) refers the diversion as a standard moral hazard problem: (i) the higher $\Phi$ also increases the gain from diversion, and the diversion opportunity cost increases with (ii) bank efficiency $p$ and (iii) the spread between the corporate lending rate $R$ and the return on the storage technology $\gamma$. The last feature, which implies that as the corporate lending rate goes down, banks have an incentive to walk away from the interbank market, is consistent with recent empirical evidence of taking high risks by the banks during low interest rates. (see, e.g., Maddaloni and Peydró, 2011; Jiménez et al., 2014).

### 3.5.3.3 Asymmetric Information

The intermediation skill of the banks are private information; ex ante lenders cannot observe them and also cannot verify them ex post. Therefore, lenders ignore any private

incentives of the borrowers to divert funds. This makes the interbank loan contracts the same for all banks, i.e., none of the market funding ratio ($\Phi$), and the interbank lending rate ($\rho$) depends on $p$. However, the lenders want to deter the borrowers from diverting. This is done by limiting the quantity of funds to the borrowers so that the least efficient banks with $p < \bar{p}$ have no incentive to pretend as high efficiency bank and divert:

$$\gamma(1 + \theta\Phi) \leq \rho \tag{3.7}$$

This is the incentive compatibility constraint which, eventually, sets a limit for $\Phi$, i.e., the market funding ratio above which no bank would like to lend. Hence, it can also be regarded as the lenders' funding tolerance. At the optimum, this incentive compatibility constraint binds and the borrowing banks utilize the full borrowing capacity:

$$\Phi \equiv \frac{\rho - \gamma}{\gamma\theta}$$

The borrowing capacity ($\Phi$) increases with the interbank lending rate ($\rho$). With a higher $\rho$, banks lend more, and only more efficient (high $p$) banks keep demanding a loan. Lenders have an incentive to tolerate a higher market funding ratio $\Phi$ since more efficient banks have lower incentive to divert. Therefore, there is a positive selection effect on the borrowers caused by an increase in the interbank rate. Symmetrically, there is a detrimental effect on incentives caused by a decrease in the interbank rate. In limit, when the interbank rate is equal to the return on storage ($\rho = \gamma$), the demand is null ($\Phi = 0$) since no borrowers can commit herself to repay.

## 3.6   Equilibrium Analyses

The interbank market equilibrium largely depends on whether the country is a closed economy or integrated with other countries. We will first characterize the equilibrium of the banking sector in one country without any integration (see Boissay et al., 2016 for full

explanations). Then, we will introduce different dimensions of integration between two countries and characterize the corresponding interbank market equilibrium.

### 3.6.1 Markets Without Integration

#### A. Interbank Market Clearing

In interbank markets equilibria of two countries without any integration, the interbank rate $\rho$ clears the markets. We seek for an equilibrium where the return on storage is lower than the interbank rate ($\gamma < \rho$) so that the trade takes place ($\Phi > 0$). In a world of two countries without any integration, a mass $\mu(\bar{p})$ of banks lend and the complement mass $1 - \mu(\bar{p})$ of banks borrow $\Phi$ per unit of deposit. The market clearing condition thus becomes

$$\underbrace{A\mu\left(\frac{\rho}{R}\right)}_{\text{Supply}} = \underbrace{A\left[1 - \mu\left(\frac{\rho}{R}\right)\right]\overbrace{\frac{\rho-\gamma}{\theta\gamma}}^{\text{Intensive Margin}}}_{\text{Demand}} \tag{3.8}$$

$$\Leftrightarrow R = \Psi(\rho) \equiv \frac{\rho}{\mu^{-1}\left(\frac{\rho-\gamma}{\rho-\gamma(1-\theta)}\right)}$$

The interbank market clearing condition shows that as the interbank rate, $\rho$ increases so does the aggregate supply, while there are two opposite forces in the aggregate demand. On the one hand, an increase in the interbank rate causes lower aggregate demand because fewer borrowers demand funds at higher rate; this is an extensive margin effect. On the other hand, a higher interbank rate increases the borrowing capacity and thus aggregate demand of funds; this is the intensive margin effect. At the aggregate level, when there are more borrowers, i.e., when $\rho$ is small enough the later effect more than offsets the earlier effect. It follows that when the interbank rate $\rho$ is small, the aggregate demand curve bends backward and increases with $\rho$. The function $\Psi(\rho)$ is strictly convex and there exists a threshold $\bar{R} \equiv \Psi(\bar{\rho})$ for the corporate loan rate $R$ above which there are two equilibria with trade and below which there is no equilibrium with trade. In the later
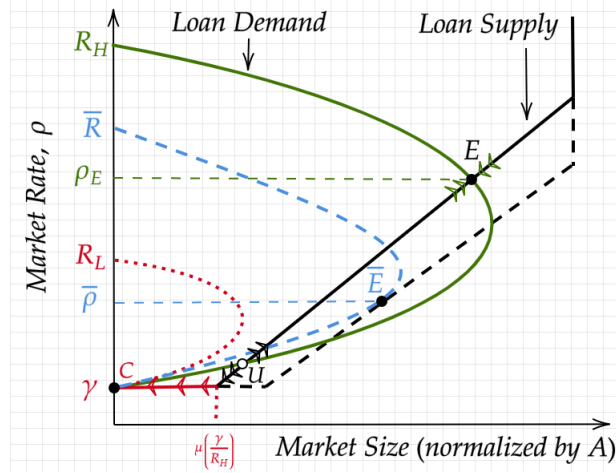
Figure 3.3: Interbank Market Clearing

case, there exists a cutoff $\bar{p} = \gamma/R$ such that banks with $p < \bar{p}$ store, banks with $p > \bar{p}$ lend to the firm, and bank $\bar{p}$ is indifferent. We will refer to such no-trade equilibrium as a banking crisis. However, such equilibrium is Pareto-dominated by the trade equilibrium and therefore we rule out this equilibrium assuming that banks always coordinate in trade equilibrium. Figure 3.3 shows the overall situation.

**B. Aggregate Corporate Loan Market Clearing**

The aggregate supply of corporate loans depends on the existence of the interbank market operations. On the one hand, when interbank trades take place, the supply of corporate loans is $A$, i.e., all bank assets are channeled to the firms. On the other hand, when the interbank market is frozen, banks with $p < \gamma/R$ use the storage technology and the aggregate supply of corporate loans is $[1 - \mu(\gamma/R)]A$. The demand of aggregate corporate loan is determined through solving for capital, $k$ from firm's profit maximization problem. Thus, the aggregate corporate loan clearing condition is

$$f_k^{-1}\left(\frac{R+\delta-1}{z}\right) = \begin{cases} A & \text{for equilibrium with trade} \\ A\left[1 - \mu\left(\frac{\gamma}{R}\right)\right] & \text{otherwise} \end{cases} \tag{3.9}$$

where $f_k(k) \equiv \partial F(k,1)/\partial k$.
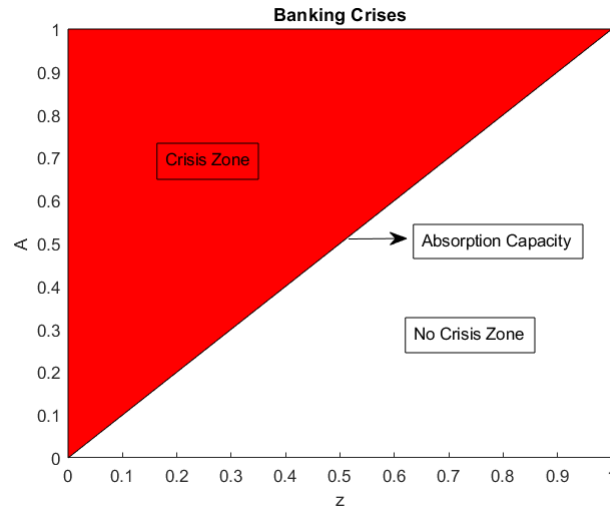
Figure 3.4: Banking Crises Zone

### C. Interbank Loan Market Freeze

The interbank loan market freezes if and only if

$$A > \overline{A} \equiv f_k^{-1}((\overline{R} + \delta - 1)/z) \Leftrightarrow z < \overline{z} \equiv (\overline{R} + \delta - 1)/f_k(A);$$

otherwise, the interbank loan market operates. $\overline{A}$ is the "absorption capacity" of the banks. This is the maximum quantity of assets that the banks can allocate efficiently. Above this threshold there is no trade in the interbank market because of the counterparty fears. This is equivalent to the TFP threshold $\overline{z}$, below which the interbank market freezes. Market freezes thus result either from the overaccumulation of assets by the household or from an adverse productivity shock that reduces banks' absorption capacity, or from a combination of both endogenous and exogenous factors. Figure 3.4 exhibits this situation.

### 3.6.2 Two firms Closed Economy Model

Let us continue to think in terms of a closed economy. There are two firms $i = 1, 2$ with identical technology and equal mass ($0.5$ each). Their productivity follows the same AR(1) process, but the shocks to productivity are independent draws. As far as the banking sector's deposit, loan, and interbank rates are concerned, they will continue to follow the relationships derived in previous sections. The wages, i.e., Marginal Productivity of Labor (MPL) across the two firms must be identical. The labor market equilibrium (with household supplying labor inelastically at $1$) is as follows:

$$z_1 f_h (k_1, h_1) = z_2 f_h (k_2, h_2)$$
$$0.5 (h_1 + h_2) = 1$$

For Cobb-Douglas technology with $\alpha$ as the capital share, capital/labor ratio across the two firms will follow

$$\frac{k_1}{h_1} = \left(\frac{z_2}{z_1}\right)^{\frac{1}{\alpha}} \frac{k_2}{h_2} \tag{3.10}$$

Since both types rent capital at the same market rate, their Marginal Productivity of Capitals must be equal.

$$\frac{k_1}{h_1} = \left(\frac{z_1}{z_2}\right)^{\frac{1}{1-\alpha}} \frac{k_2}{h_2} \tag{3.11}$$

However, (3.10) and (3.11) can not hold simultaneously. Thus only the high productivity firm will produce. It will rent all the capital and hire all the labor available in the economy. The equilibrium is trivially the same as in Boissay et al. (2016), except that $\max \{z_1, z_2\}$ will be the economy's productivity.

### 3.6.3 A Two-country World with Perfect Integration

In a closed economy model the monitoring costs $(1 - p) R$ are rebated back to the households to abstract from wealth effects of monitoring costs. In a two-country world,

68

where a country's savings may end up for deposit and loans in another country, we will assume that these transfers go to the source of the savings. Suppose all capital markets and labor markets are integrated in a symmetric two-country world. The environment is then equivalent to a closed economy with two firms. Only one country will produce by renting the entire world's capital and hiring the total labor supply of the two countries. This case is obviously not interesting.

### 3.6.4 A Two-country World without Labor Mobility

In this section, we assume that households can work only at the domestic firm. Under this restriction, firms in both countries will operate and produce. Under integration, deposits made by household, deposits taken by banks, and loans made by banks may differ. Let $A_j$ continue to be the aggregate deposits made by households of country $j$, let $D_j$ denote the deposits taken by a (ex-ante identical) bank of country $j$, and $k_j$ denote the loan (that equals the capital stock) taken by the representative firm in country $j$.

We will alternatively consider environments in which cross-border deposits are (1) permitted or (2) prohibited. In each of these, we will in turn consider cases in which (i) cross-border interbank as well as corporate loans are permitted, (ii) only corporate loans are permitted and (iii) only interbank lending is permitted.

#### 3.6.4.1 With Cross-border Deposit Mobility

In this subsection, we assume that the depositors and the banks in either country are free to make and take cross-border deposits.

**a (i): With Open Interbank and Corporate Loan Markets**

In this case, banks can lend to both banks and corporates of either country. From the perspective of depositors, all banks are then multinational and ex-ante identical. The total

deposits of the two countries $A_1 + A_2$ will be equally deposited at all the banks, now with a mass of 2. Therefore the deposits taken by each bank will be

$$D_j = D = \frac{A_1 + A_2}{2} \tag{3.12}$$

Under interbank trade the interest rate $\rho$ will be common across the two countries. Furthermore, the firms' borrowing rate $R$ will also be common. Then, with inelastic labor supply fixed at unity, the two countries' capital ratios will follow from (3.11) as

$$k_1 = \left(\frac{z_1}{z_2}\right)^{\frac{1}{1-\alpha}} k_2 \tag{3.13}$$

The market clearing condition for the corporate loan market can be expressed as

$$\sum_j k_j = \sum_{j=1,2} f_{k_j}^{-1}\left(\frac{R+\delta-1}{z_j}\right) = \begin{cases} \sum_j A_j & \text{for trade equilibrium} \\ \left[1 - \mu\left(\frac{\gamma}{R}\right)\right]\sum_j A_j & \text{otherwise} \end{cases} \tag{3.14}$$

where the first case holds if and only if $R > \overline{R} > \Psi(\bar{\rho})$.

When interbank trade exists, the total supply of interbank loans will equal the total demand:

$$2D\mu\left(\frac{\rho}{R}\right) = 2D\left[1 - \mu\left(\frac{\rho}{R}\right)\right]\frac{\rho-\gamma}{\theta\gamma} \tag{3.15}$$

**a (ii): With Open Corporate Loan Market Only**

Let us continue to assume that household can deposit in either country's banks. While banks cannot make cross-border interbank loans, the firms can still borrow from banks across the border. Banks can still lend to other domestic banks and therefore the interbank loan market equilbrium must satisfy

$$D\mu\left(\frac{\rho_j}{R}\right) = D\left[1 - \mu\left(\frac{\rho_j}{R}\right)\right]\frac{\rho_j-\gamma}{\theta\gamma}.$$

where we have incorporated the fact that with open corporate loan market, $R$ is common across the two countries. The above in turn implies that $\rho$ will also be common and so will

be the per unit deposit rate. Once again, from the perspective of the depositors, banks in the two countries are fullly multinational. As in the previous case with complete banking sector integration, the deposits are

$$D_j = D = \frac{A_1 + A_2}{2}$$

The two countries' capital ratios will follow from (3.11) as

$$k_1 = \left(\frac{z_1}{z_2}\right)^{\frac{1}{1-\alpha}} k_2$$

The market clearing condition for the corporate loan market can be expressed as

$$\sum_j k_j = \sum_{j=1,2} f_{k_j}^{-1}\left(\frac{R+\delta-1}{z_j}\right) = \begin{cases} \sum_j A_j & \text{for trade equilibrium} \\ \left[1-\mu\left(\frac{\gamma}{R}\right)\right]\sum_j A_j & \text{otherwise} \end{cases}$$

where the first case holds if and only if $R > \overline{R} > \Psi(\bar{\rho})$.

**a (iii): With Open Interbank Market Only**

Let us continue to assume that household can deposit in either country's banks. While banks can make interbank loans internationally, the firms can only borrow from their own country's banks. The banks are no-longer multinationals – their opportunity sets now diverge. Hence, the deposits they attract are no-longer going to be identical. They will be determined by the equilibrium as described below.

First note that two arbitrage conditions must hold. First, deposit rates offered by banks must be equal across the two countries. Second, under interbank trade, no storage will occur in either country.

**Lemma 1** *In an interbank trade equilibrium the corporate loan rates in both countries are equalized.*

**Proof.** Let $\Phi_j \geq 0$ be the publicly known interbank borrowing (per unit of deposit) in country $j$. Irrespective of $\Phi_j$, the cutoff rule for the marginal lender/borrower bank is:

$$\bar{p}_j = \frac{\rho}{R_j}.$$

Since household can deposit in either country, all banks (in both countries) must offer the same deposit rate:

$$r = R_j \int_{\frac{\rho}{R_j}} p \frac{d\mu(p)}{1 - \mu(p)}$$

Hence, $R_j = R$ for both country $j$. This in turn requires that

$$k_1 = \left(\frac{z_1}{z_2}\right)^{\frac{1}{1-\alpha}} k_2$$

Since no storage occurs under interbank trade

$$k_1 + k_2 = A_1 + A_2$$

Finally, since the incentive compatibility constraint holds with equality, i.e., $\gamma(1 + \theta\Phi) = \rho$, in both countries the borrowing capacity, $\Phi = \frac{\rho - \gamma}{\gamma\theta}$ must be the same. Within each country the mass of banks lending in the interbank market, $\mu(\frac{\rho}{R})$, is the same. Also the borrowing per unit of deposits for the banks making corporate loans $\Phi$ is the same. This implies that

$$k_j = D_j$$

Banks in each country attract an amount of deposit that clears the domestic interbank lending as well as corporate loans. Hence, once the deposits are made, there is no need for cross-border interbank or corporate loans.

The banking sector equilibrium within each country follows the closed economy, which leads to the following proposition

**Proposition 1** *Interbank trade exists in both countries if and only if*

$$\sum_{j=1,2} f_{k_j}^{-1}\left(\frac{\overline{R} + \delta - 1}{z_j}\right) \geq \sum_{j=1,2} A_j$$

The converse of this result is that a banking sector freeze must occur simultaneously across the two countries. However, the deposit rates will still be the same across the two

countries. This is only possible if

$$r = R_j \left[ \frac{\gamma}{R_j} \mu \left( \frac{\gamma}{R_j} \right) + \int_{\frac{\gamma}{R_j}} p \frac{d\mu(p)}{1 - \mu(p)} \right] \text{ for both country } j.$$

Once again $R_j = R$ for $j = 1, 2$ must hold and the capital used by firms must follow (3.13).

**Corollary 1** *Under a banking sector freeze, the equilibrium capital allocations and corporate loan rate* $\{k_1, k_2, R\}$ *are given by*

$$\sum_j k_j = \sum_{j=1,2} f_{k_j}^{-1} \left( \frac{R + \delta - 1}{z_j} \right) = \left[ 1 - \mu \left( \frac{\gamma}{R} \right) \right] \sum_j A_j;$$

$$k_1 = \left( \frac{z_1}{z_2} \right)^{\frac{1}{1-\alpha}} k_2.$$

Finally, in an interbank trade equilibrium, the equilibrium rate $\rho(R)$ continues to be determined by (3.15) with $2D$ on its both sides now trivially replaced by $\sum D_j$:

$$\left( \sum_{j=1,2} D_j \right) \mu \left( \frac{\rho}{R} \right) = \left( \sum_{j=1,2} D_j \right) \left[ 1 - \mu \left( \frac{\rho}{R} \right) \right] \frac{\rho - \gamma}{\theta \gamma} \tag{3.16}$$

### 3.6.4.2   Without Cross-border Deposit Mobility

Banks in each country now take only domestic deposits:

$$D_j = A_j. \tag{3.17}$$

However, their asset side has no cross-border restrictions. We now sequentially consider the three alternatives.

#### b (i): With Open Interbank and Corporate Loan Markets

The banking sector equilibrium in this case is fairly similar to that under case a(i) except for the size of the banks' balance sheets. Banks now have access to only domestic pool of deposits. The amount of loans therefore have to balance accordingly. With common corporate loan rates in the two countries:

$$k_1 = \left( \frac{z_1}{z_2} \right)^{\frac{1}{1-\alpha}} k_2$$

must hold. However, unlike in case a(i) $k_j \neq D_j = A_j$ now. That is, firms in country $j$ may be borrowing from the other country and vice versa. When interbank trade exists, a common $\rho = \Psi^{-1}(R)$ emerges from (3.16)

$$\left(\sum_{j=1,2} A_j\right) \mu\left(\frac{\rho}{R}\right) = \left(\sum_{j=1,2} A_j\right)\left[1 - \mu\left(\frac{\rho}{R}\right)\right]\frac{\rho - \gamma}{\theta\gamma}$$

Finally, the world-economy will jointly experience a credit crunch iff

$$\sum_{j=1,2} f_{k_j}^{-1}\left(\frac{\overline{R} + \delta - 1}{z_j}\right) < \sum_j A_j,$$

where $\overline{R} = \Psi(\bar{\rho})$. The corporate loan market clearing can be expressed as

$$\sum_j k_j = \sum_{j=1,2} f_{k_j}^{-1}\left(\frac{R + \delta - 1}{z_j}\right) = \begin{cases} \sum_j A_j & \text{for trade equilibrium} \\ \left[1 - \mu\left(\frac{\gamma}{R}\right)\right]\sum_j A_j & \text{otherwise} \end{cases}$$

where the first case holds if $R > \Psi(\bar{\rho})$.

### b (ii): With Open Corporate Loan Market Only

Here, banks can make cross-border loans to firms and therefore $R$ must be equal in the two countries. Banks can make within country interbank loans. Let $\rho_j$ denote within country interbank rates. The interbank loan market equilibrium can now be expressed as

$$A_j \mu\left(\frac{\rho_j}{R}\right) = A_j\left[1 - \mu\left(\frac{\rho_j}{R}\right)\right]\frac{\rho_j - \gamma}{\theta\gamma}.$$

Obviously, $\Psi : \rho \to R$ is common for both countries.

**Proposition 2** *With open corporate loan markets, within country interbank lending rates are equal.*

With common corporate loan rates in the two countries:

$$k_1 = \left(\frac{z_1}{z_2}\right)^{\frac{1}{1-\alpha}} k_2$$

74

must hold. However, as in b(i) $k_j \neq A_j$. As in b(i), the world-economy will jointly experience a credit crunch iff

$$\sum_{j=1,2} f_{k_j}^{-1}\left(\frac{\overline{R}+\delta-1}{z_j}\right) < \sum_j A_j,$$

where $\overline{R} = \Psi(\bar{\rho})$. The corporate loan market clearing can be expressed as

$$\sum_j k_j = \sum_{j=1,2} f_{k_j}^{-1}\left(\frac{R+\delta-1}{z_j}\right) = \begin{cases} \sum_j A_j & \text{for trade equilibrium} \\ \left[1-\mu\left(\frac{\gamma}{R}\right)\right]\sum_j A_j & \text{otherwise} \end{cases}$$

where the first case holds if $R > \Psi(\bar{\rho})$. In the second case, the amount stored in each country equals $\mu\left(\frac{\gamma}{R}\right) A_j$.

**b (iii): With Open Interbank Market Only**

Under interbank trade, once again, $\rho$ will be common in both countries. However, corporate loan rates may no longer be equal. The interbank loan market equilibrium can now be expressed as

$$\underbrace{A_1\mu(\bar{p}_1)}_{\text{Domestic Supply}} + \underbrace{A_2\mu(\bar{p}_2)}_{\text{Foreign Supply}} = \underbrace{A_1[1-\mu(\bar{p}_1)]\Phi_1}_{\text{Domestic Demand}} + \underbrace{A_2[1-\mu(\bar{p}_2)]\Phi_2}_{\text{Foreign Demand}}$$

Using $\Phi_1 = \Phi_2 = \Phi = \frac{\rho-\gamma}{\gamma\theta}$, and $\bar{p}_j = \frac{\rho}{R_j}$, the market clearing condition becomes,

$$A_1\mu\left(\frac{\rho}{R_1}\right) + A_2\mu\left(\frac{\rho}{R_2}\right) = \left[A_1\left(1-\mu\left(\frac{\rho}{R_1}\right)\right) + A_2\left(1-\mu\left(\frac{\rho}{R_2}\right)\right)\right]\frac{\rho-\gamma}{\theta\gamma} \quad (3.18)$$

$$\Leftrightarrow \frac{A_1}{A_2} = -\frac{\mu_2(1+\Phi)-\Phi}{\mu_1(1+\Phi)-\Phi}$$

where, $\mu_j = \mu\left(\frac{\rho}{R_j}\right)$ for $j \in \{1,2\}$.

It is not possible to characterize the properties of $R_1$ and $R_2$ as a function of $\rho$. We need to characterize $\rho$ as a function of $R_1$ and $R_2$. It further complicates the multiplicity problem by perhaps increasing its dimensionality. One may have to address this numerically. Suppose a solution exists such that $R_1, R_2 > \rho > \gamma$. In equilibrium

$$k_j = f_{k_j}^{-1}\left(\frac{R_j+\delta-1}{z_j}\right), \text{ for } j = 1,2, \quad (3.19)$$

and the market clearing for corporate loans requires supply of corporate loans, $l_j$ be

$$l_j = A_j \left[1 - \mu\left(\frac{\rho}{R_j}\right)\right]\left(1 + \frac{\rho - \gamma}{\theta\gamma}\right), \text{ for } j = 1, 2. \tag{3.20}$$

Under a closed economy the aggregate supply of corporate loans in normal times would be the total deposit collected by the banks. However, equation (3.20) is a novel finding of the model where the aggregate corporate lending does not only depend on the savings of the households, i.e., $A$ but on other factors- such as- corporate lending rate-$R$, interbank lending rate- $\rho$, and storage components- $\gamma$ and $\theta$ as well.

There are now five unknowns in the model: $\{R_1, R_2, k_1, k_2, \rho\}$. If an equilibrium exists, they should sensibly solve the five equations: (3.18), (3.19), and (3.20).

If there is no solution to the above problem, then we are back in the crisis world. Both countries will be in autarky with their capital employment and the loan rates given by the two equations:

$$l_j = \left[1 - \mu\left(\frac{\gamma}{R_j}\right)\right]A_j = f_{k_j}^{-1}\left(\frac{R_j + \delta - 1}{z_j}\right), \text{ for } j = 1, 2$$

while the amount stored in each country equals $\mu\left(\frac{\gamma}{R_j}\right)A_j$.

**Banking sector's return on deposit:** The return on deposits depends on three factors: corporate loan revenue, interbank lending revenue, and the borrowing cost. These factors are as follows for country $j = 1, 2$:

Corporate loan revenue: $\int_{\bar{p}_j}^1 p\left[R_j\left(1 + \Phi\right)\right]d\mu(p)$

Interbank loan revenue: $\int_0^{\bar{p}_j} p\rho d\mu(p)$

Interbank borrowing cost: $\int_{\bar{p}_j}^1 p\rho\Phi d\mu(p)$

The banking sector's return on deposit would be:

$$r_j \equiv \int_0^1 r_j(p)d\mu(p)$$

$$= \begin{cases} R_j\left[\int_{\bar{p}_j}^1 p(1+\Phi)d\mu(p)\right] + \int_0^{\bar{p}_j} p\rho d\mu(p) - \int_{\bar{p}_j}^1 p\rho\Phi d\mu(p) & \text{for equilibrium with trade} \\ R_j\left[\int_{\gamma/R_j}^1 pd\mu(p)\right] + \gamma\mu\left(\frac{\gamma}{R_j}\right) & \text{otherwise} \end{cases}$$

Rearranging this, we have

$$
r_j = \begin{cases} \int_0^{\bar{p}_j} p\rho d\mu(p) + \int_{\bar{p}_j}^1 p\left[R_j + \Phi(R_j - \rho)\right] d\mu(p) & \text{for equilibrium with trade} \\ R_j \left[\int_{\gamma/R_j}^1 p d\mu(p)\right] + \gamma\mu\left(\frac{\gamma}{R_j}\right) & \text{otherwise} \end{cases}
\tag{3.21}
$$

where under the equilibrium trade, the first term represents the interbank lending revenue of a $\mu(\bar{p}_j)$ mass of banks those are lenders and the second term denotes the corporate loan revenue plus profit from market funded capital for a $1 - \mu(\bar{p}_j)$ mass of banks those are borrowers. In contrast, when there is no equilibrium exists with trade, there is no banking intermediation, and a mass $\mu(\gamma/R_j)$ of banks find it optimal to use the storage technology.

### 3.6.5 General Equilibrium

Before we proceed to the quantitative analysis, a general equilibrium of the world economy in which two countries connected only with open interbank market is defined as follows. The state variables for a particular individual's optimization problem in country $j$ are $(i)$ the individual asset holdings $a_j$, $(ii)$ the aggregate asset holdings $\{A_1, A_2\}$, and $(iii)$ the realization of the technology shocks $\{z_1, z_2\}$. Let $s = \{A_1, A_2, z_1, z_2\}$. In the sequel, we denote by $\Gamma_j(s)$ the perceived law of motion of aggregate assets in the country $j$ and by $R_j(s), r_j(s)$, and $\rho(s)$ and $w_j(s)$ the pricing functions for corporate loans, deposits, interbank loans and labor, respectively; we also denote by $\pi_j(s)$ and $\chi_j(s)$ the profit and rebate functions. All these functions are function of aggregate assets $\{A_1, A_2\}$ and productivity $\{z_1, z_2\}$, which are both taken as given by the household and capture the presence of externalities. The household $j$'s recursive optimization problem written as

$$
V(a, s) = \max_{a', c, n} u(c - \nu(n)) + \beta \mathbb{E}_t V(a', s')
\tag{3.22}
$$

subject to

$$a' + c = r(s)a + w(s)n + \pi(s) + \chi(s)$$

$$A' = \Gamma(s)$$

where, in the calibrated version of the model, $u(x) = x^{1-\sigma}/(1-\sigma)$ and $\nu(n) = n^{1+\nu}/(1+\nu)$. The solution to this problem is a set of decision rules $a(a_j, s), n(a_j, s)$, and $c(a_j, s)$.

The firm's problem in country $j$ is simply given by

$$\max_{k,h} zF(k, h) + (1 - \delta)k - w(s)h - R(s)k,$$

which leads to the decision rules $k(s)$ and $h(s)$. Finally, the solution of banks' problem in country $j$ leads to the aggregate loans $l(s)$ and $\phi(s) = (\rho(s) - \gamma)/\gamma\theta$, where only aggregate assets enter the solution due to the linearity of the problem. In the recursive rational expectation equilibrium, actual and perceived law of motions coincide in each country $j$, respectively.

### 3.6.5.1  *Definition (Recursive competitive general equilibrium)*

*A recursive competitive equilibrium of the world economy, given the aggregate state $s = \{A_1, A_2, z_1, z_2\}$, is a sequence of prices defined by the pricing functions $R(s), r(s), \rho(s)$, and $w(s)$, two perceived law of motions for aggregate assets $\Gamma_1(s)$ and $\Gamma_2(s)$, and a set of decision rules for each country $j = 1, 2$ of $\{c(a_j, s), a'(a_j, s), k(a_j, s), h(a_j, s), \phi(a_j, s), l(a_j, s)\}$ with the value function $V(a_j, s)$ such that,*

1. *$\{c(a_j, s), a'(a_j, s), n(a_j, s)\}$ and $V(a_j, s)$ solve the household j's recursive optimization problem taking $R_j(s), r_j(s), w_j(s), \pi_j(s), \chi_j(s)$ and $\Gamma_j(s)$ as given.*

2. *$k_j(s)$ and $h_j(s)$ solve the firm j's optimization problem taking $R_j(s), w_j(s)$ as given.*

3. $\phi_j(s)$ *solves the banks' optimization problem taking* $R_j(s), r_j(s),$ *and* $\rho(s)$ *as given. Aggregate loans in each country j are, respectively*

$$l_j(s) = \begin{cases} A_j \left[1 - \mu\left(\frac{\rho}{R_j(s)}\right)\right]\left(1 + \frac{\rho-\gamma}{\theta\gamma}\right), & \text{if } A \leq \bar{A}(z_1, z_2) \\ \left[1 - \mu\left(\frac{\gamma}{R_j(s)}\right)\right]A_j, & \text{otherwise} \end{cases}$$

4. *The perceived law of motion for aggregate assets is consistent with the actual law of motions:*

$$a'_j(a, s) = \Gamma_j(s).$$

5. *Wages satisfy* $w_j(s) = z_j F_h(k_j(s), h_j(s))$, *and the corporate loan rate satisfies* $R_j = z_j F_k(k_j(s), h_j(s)) + 1 - \delta$, *the deposit rate satisfies*

$$r_j(s) = \begin{cases} \int_0^{\bar{p}_j(s)} p\rho d\mu(p) + \int_{\bar{p}_j(s)}^1 p\left[R_j(s) + \Phi(R_j(s) - \rho)\right]d\mu(p) & \text{if } A \leq \bar{A}(z_1, z_2) \\ R_j(s)\left[\int_{\gamma/R_j(s)}^1 p d\mu(p)\right] + \gamma\mu\left(\frac{\gamma}{R_j(s)}\right) & \text{otherwise} \end{cases}$$

*where* $\bar{p}_j(s) = \rho(s)/R_j(s)$

6. *The aggregate intermediation cost rebated to the household is given by* $\chi_j(s) = (R_j(s) - r_j(s))A_j - (R_j(s) - \gamma)(A_j - k_j(s))$, *and the firm j's profits are equal to zero,* $\pi_j(s) = 0$.

7. *Goods, labor, capital, and interbank market clear:*

$$c_j(A_j, s) + a'_j(A_j, s) = z_j F(k_j(s), h_j(s)) + (\gamma + \delta - 1)(A_j - k_j(s)) + (1 - \delta)A_j$$

$$h_j(s) = n_j(A_j, s)$$

$$k_j(s) = l_j(s)$$

$$\sum_{j=1,2} A_j \mu\left(\frac{\rho(s)}{R_j(s)}\right) = \sum_{j=1,2} A_j \left[1 - \mu\left(\frac{\rho(s)}{R_j(s)}\right)\right]\frac{\rho(s) - \gamma}{\theta\gamma}$$

8. *The banking sector's absorption capacity is given by*

$$\bar{A}(z_1, z_2) = \left(\frac{1-\alpha}{\vartheta}\right)^{\frac{1}{\nu}}\left(\frac{\alpha}{\bar{R} + \delta - 1}\right)^{\frac{\nu+\alpha}{\nu(1-\alpha)}}\sum_{j=1,2} z_j^{\frac{1+\nu}{\nu(1-\alpha)}}$$

## 3.7   Quantitative Analysis

In this section, we investigate the quantitative properties of the interbank market equilibrium of the model in two parts. First, we set the functional forms and the parameters to calibrate the interbank market equilibrium. Second, we investigate the results on the equilibrium for different possible states. Following Boissay et al. (2016), we investigate the quantitative properties of the general equilibrium model using the calibrated values of the parameters from Table 1.

### 3.7.1   Calibration

The technology of the representative firm production is represented by a constant returns to scale production function of the form $z_j F(k_j, h_j) \equiv z_j k_j^\alpha h_j^{1-\alpha}$ with $\alpha \in (0, 1)$ for $j = 1, 2$. The capital elasticity $\alpha$ is set to $0.3$ in the production function, and we assume that the capital depreciation rate is 10 percent per annum ($\delta = 0.1$). We also assume the persistence of the productivity shock $\rho_z = 0.89$ and standard deviation of productivity shock $\sigma_z = 0.013$ or $1.3$ percent following Boissay et al. (2016). For the Foreign country, we use the AR(1) estimates that are consistent with those of the peripheral European countries so that $\sigma_{zf} = 3$ percent. We solve the model numerically using the collocation method, with decision rules iterations as in Boissay et al. (2016), allowing for discontinuities in the asset accumulation at the points at which the economy switches regime.

The banking sector parameters include the distribution of banks $\mu(\cdot)$, the diversion technology parameter, $\theta$, and the return on storage $\gamma$. We assume that $\mu(p) = p^\lambda$ with $\lambda \in \mathbb{R}^+$. The other banking parameters are calibrated jointly so that a financial recession occurs rarely (on average every 42 years as in fact 1) and satisfying other targets as in Boissay et al. (2016). Table 1 has the calibration parameters for the interbank market equilibrium.

Table 3.1: Calibration Parameters

| Parameters | Variable | Value |
|---|---|---|
| Discount factor (Deflated for growth) | $\beta$ | 0.971 |
| Inverse of Frisch elasticity | $\nu$ | 0.500 |
| Labor disutility | $\vartheta$ | 0.945 |
| Risk aversion | $\sigma$ | 2.000 |
| Capital elasticity | $\alpha$ | 0.300 |
| Capital depreciation rate | $\delta$ | 0.100 |
| Std. of productivity shock in Home Country | $\sigma_{zh}$ | 0.013 |
| Std. of productivity shock in Foreign Country | $\sigma_{zf}$ | 0.030 |
| Persistence of productivity shock | $\rho_z$ | 0.890 |
| Bank distribution; $\mu(p) = p^\lambda$ | $\lambda$ | 26.00 |
| Diversion cost | $\theta$ | 0.085 |
| Storage technology | $\gamma$ | 0.952 |

### 3.7.2 Results

To solve for the five unknowns $(R_1, R_2, k_1, k_2, \rho)$, we need to work with five equations: (3.18), (3.19), and (3.20). The first equation is interbank equilibrium:

$$A_1 \mu \left( \frac{\rho}{R_1} \right) + A_2 \mu \left( \frac{\rho}{R_2} \right) = \left[ A_1 \left( 1 - \mu \left( \frac{\rho}{R_1} \right) \right) + A_2 \left( 1 - \mu \left( \frac{\rho}{R_2} \right) \right) \right] \frac{\rho - \gamma}{\theta \gamma}$$

Replacing $\mu(p) = p^\lambda$, we get

$$A_1 \left( \frac{\rho}{R_1} \right)^\lambda + A_2 \left( \frac{\rho}{R_2} \right)^\lambda = \left[ A_1 \left( 1 - \left( \frac{\rho}{R_1} \right)^\lambda \right) + A_2 \left( 1 - \left( \frac{\rho}{R_2} \right)^\lambda \right) \right] \frac{\rho - \gamma}{\theta \gamma} \qquad (3.23)$$

The second and third equations are capital allocations:

$$k_j = f_{k_j}^{-1} \left( \frac{R_j + \delta - 1}{z_j} \right), \text{ for } j = 1, 2$$

Replacing the production function $F(k, 1) = k^\alpha$ we get

$$k_j = \left( \frac{R_j + \delta - 1}{\alpha z_j} \right)^{1/(\alpha - 1)}, \text{ for } j = 1, 2 \qquad (3.24)$$

The fourth and fifth equations are supply of corporate loans:

$$k_j = A_j \left[ 1 - \mu \left( \frac{\rho}{R_j} \right) \right] \left( 1 + \frac{\rho - \gamma}{\theta \gamma} \right), \text{ for } j = 1, 2$$

Replacing values of $k_j$ from (3.24) and $\mu(p) = (p)^\lambda$ we get

$$\left(\frac{R_j + \delta - 1}{\alpha z_j}\right)^{1/(\alpha-1)} = A_j\left[1 - \left(\frac{\rho}{R_j}\right)^\lambda\right]\left(1 + \frac{\rho - \gamma}{\theta\gamma}\right), \text{ for } j = 1, 2. \qquad (3.25)$$

Now, we have three unknowns: $R_1, R_2$, and $\rho$, in three nonlinear equations (3.23) and (3.25).

In case of interbank market freeze, interbank rate $\rho$ is equal to the storage return $\gamma$ and the corporate loan rates $R_j$ comes from

$$\left[\frac{R_j + \delta - 1}{\alpha z_j}\right]^{1/(\alpha-1)} = \left[1 - \left(\frac{\gamma}{R_j}\right)^\lambda\right]A_j, \quad \text{for } j = 1, 2 \qquad (3.26)$$

Since under financial integration (open interbank only with no deposit mobility) the state space includes $A_1$, $A_2$, $z_1$, and $z_2$, we can fix asset holdings and productivity of country 1 to see what happens to the solution at one point at a time. By fixing these at an average value we get similar results like Boissay et al. (2016), i.e., given asset accumulation $A$, if the productivity $z$ goes down below a threshold then the country gets into banking crisis. The country also gets into crisis if for a given productivity level $z$, a country saves beyond a threshold level. These results are visible from figure 3.5.

In figure 3.5, the red crosses are the crisis zones, i.e., either productivity is very low given the asset level or/and asset accumulation is very high given a below average productivity level. Here, we assume that the asset level and productivity level of country 1 are at an average level.

Figure 3.6 exhibits the interbank market equilibrium given fixed asset holding and productivity for country 1, and fixed asset holding but varying productivity level of country 2. We have fixed the asset holding of both countries at an average level but varied the productivity level to identify in which cases we have an interbank market operation and in which cases there are market freezes. The dotted line is for low productivity in both countries where we observe no interbank market operation. This result is intuitive because low productivity in both countries implies low requirement of capital while saving
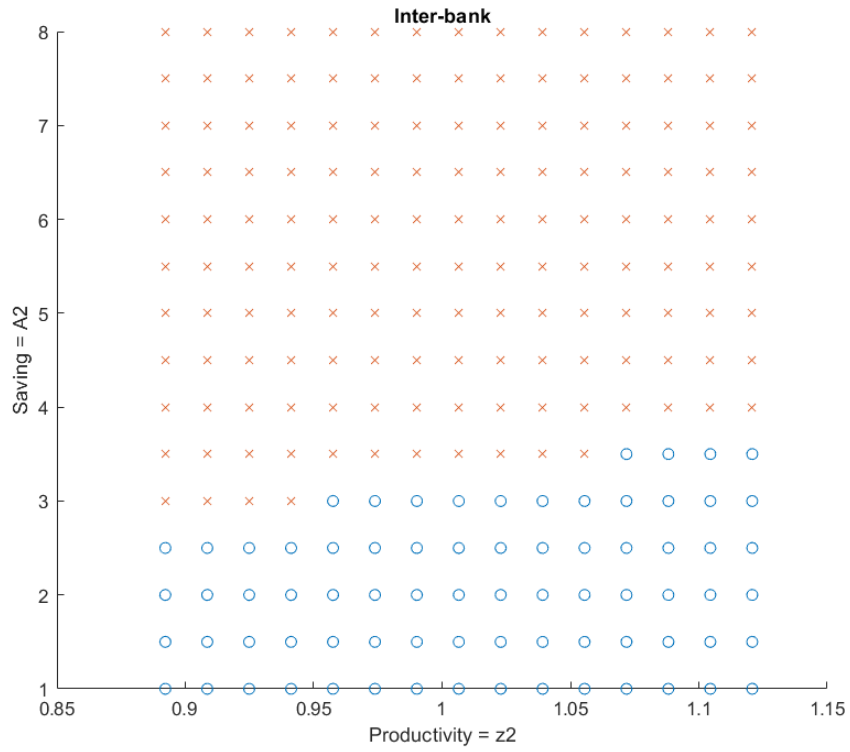
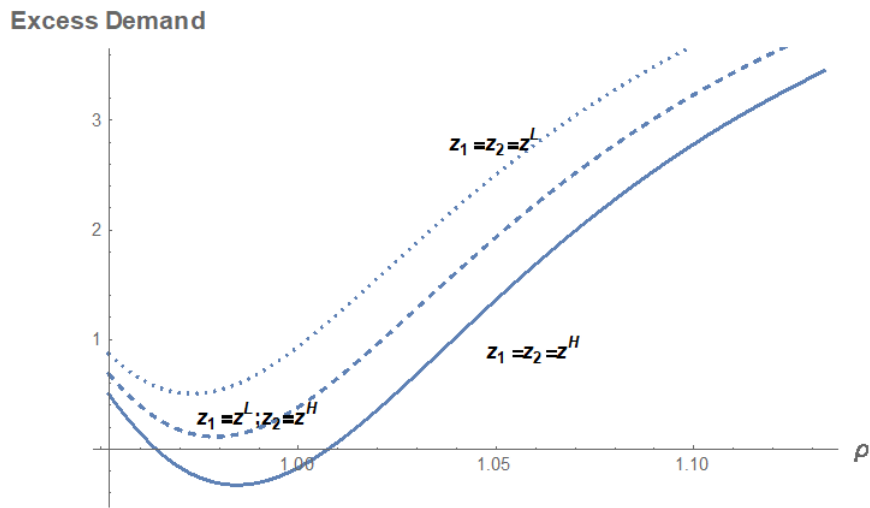Figure 3.5: Crisis and No Crisis Zone



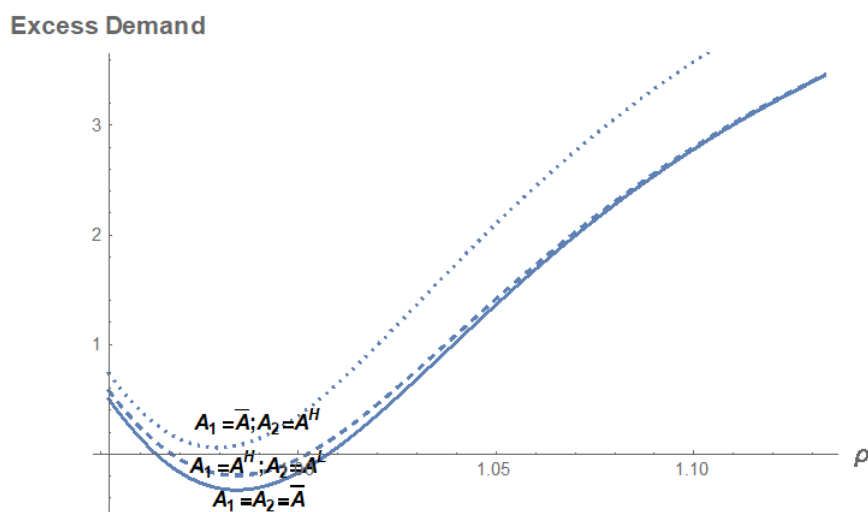Figure 3.6: Interbank Market Equilibrium with varying productivity

Figure 3.7: Interbank Market Equilibrium with varying saving

is relatively high. A higher supply with lower demand for corporate loans causes a reduction in the corporate loan rate such that no interbank lending occurs. The solid line is for high productivity in both countries where there is interbank market operation. However, we have multiple equilibria for such high productivity in both countries. The dashed line is for low productivity in country 1 but high in country 2 and we observe market freeze in this case. This is the most interesting and a key finding of this chapter . With sustainable saving and high productivity, country 2 could have an interbank trade equilibrium without any integration with country 1. However, since it has decided to "sail together", now it must also "sink together" under no interbank trade equilibrium in the integrated world.

Figure 3.7 shows the interbank market equilibrium with fixed average productivity for both countries but varying saving level. The solid line is for average saving level of both countries where there is multiple equilibria with interbank trade. The dashed line has high saving for country 1 but low saving for country 2. We have multiple trade equilibria in this case as well. However, the difference is in this time country 1 can survive

even with high level of saving because it decided to "sail together" with country 2 and their extreme end saving levels average outed. This is the positive outcome of integration which was the motive for integration. However, in the dotted line country 1 has an average asset holding while country 2 has high level of saving and there is no trade equilibrium. Country 1 could have an interbank trade equilibrium without any integration with country 2 but because they integrated they have to experience a "sink together" situation and an interbank market freeze. This is the negative outcome of the integration.

Tables 3.2 and 3.3 show the corporate lending rates in the home country ($R_1$) and the foreign country ($R_2$), respectively, with respect to different levels of asset accumulation ($A_2$) and productivity ($z_2$) in foreign country but same average levels in the home country. With low level of asset accumulation and increasing productivity level in the foreign country, the relative demand for capital rises by the firms in the foreign country. This requires an interbank funds flow from the home country to the foreign country. Therefore, the corporate lending rate goes down in the home country and goes up in the foreign country. This is also true for medium level of asset accumulation in the foreign country.

Table 3.2: $R_1$ (in net percentage rate) with varying $A_2$ and $z_2$

|  |  | $z_2$ | | |
| --- | --- | --- | --- | --- |
|  |  | Low | Medium | High |
|  | *Low* | 1.64 | 1.60 | 1.58 |
| $A_2$ | *Medium* | 2.09 | 1.95 | 1.83 |
|  | *High* | 3.00 | 3.00 | 2.33 |

Table 3.4 shows that the interbank lending rate ($\rho$) also goes up for increasing productivity in the foreign country ($z_2$), given low or medium level asset accumulation in the

foreign country and average level of asset accumulation ($A_1$) and productivity ($z_1$) in the home country due to increased overall productivity and capital demand. However, when foreign country accumulates high level of asset, the interbank market only works when the foreign country also has high level of productivity.

Table 3.3: $R_2$ (in net percentage rate) with varying $A_2$ and $z_2$

|  |  | $z_2$ |  |  |
|---|---|---|---|---|
|  |  | Low | Medium | High |
|  | *Low* | 2.73 | 3.79 | 4.87 |
| $A_2$ | *Medium* | 0.62 | 1.26 | 1.93 |
|  | *High* | 3.00 | 3.00 | 0.70 |

Table 3.4: $\rho$ (in gross rate) with varying $A_2$ and $z_2$

|  |  | $z_2$ |  |  |
|---|---|---|---|---|
|  |  | Low | Medium | High |
|  | *Low* | 0.971 | 0.974 | 0.976 |
| $A_2$ | *Medium* | 0.965 | 0.967 | 0.970 |
|  | *High* | 0.950 | 0.950 | 0.965 |

It is also evident from the tables 3.2 and 3.3 that for given productivity in the foreign country ($z_2$) and asset accumulation and productivity in the home country ($A_1, z_1$), if asset accumulation by the foreign country goes up from low to medium then, the corporate lending rate in the foreign country ($R_2$) goes down for overall increase in the supply of
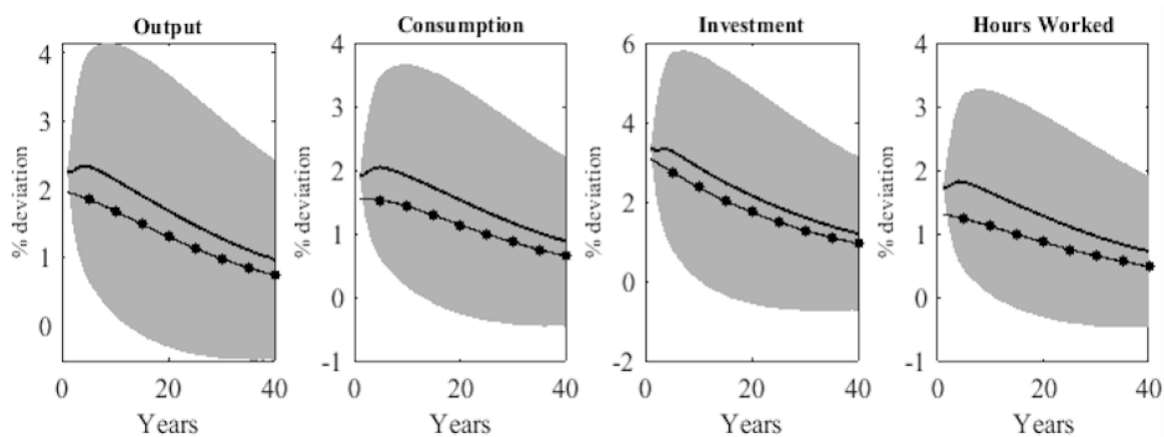
Figure 3.8: Impulse response to a one standard deviation technology shock

capital. Table 3.4 shows that this also reduces the interbank lending rate ($\rho$). Due to the equilibrium effect, the corporate lending rate in the home country ($R_1$) goes up. However, with high level of asset accumulation in the foreign country the intrebank lending market goes into a freeze since the total accumulation of two countries surpasses the total absorption capacity.

We assume the economy is at the steady state with the long-run average of $z$ to analyze the home country response. Figure 3.8 compares the dynamics in the model (thick plain line) with those in a frictionless model (round markers). It is evident from the figure that output, consumption, investment, and hours worked are comparatively larger than those in Boissay et al. (2016). Therefore, a two-country model responds more than the closed-economy model or the frictionless RBC model due to a larger interbank market integrating banking sectors of two countries together.

Figure 3.9 and 3.10 shows the typical path to financial recessions of the Home country and Foreign country, respectively. These are depicted from the simulation of the model over 50,000 periods with the years of recessions and crises identified. By convention, period 0 corresponds to the period when the financial recession (or the crises) bursts.
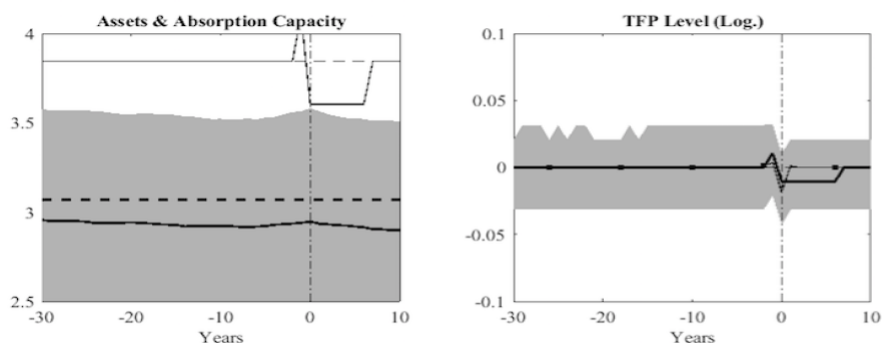
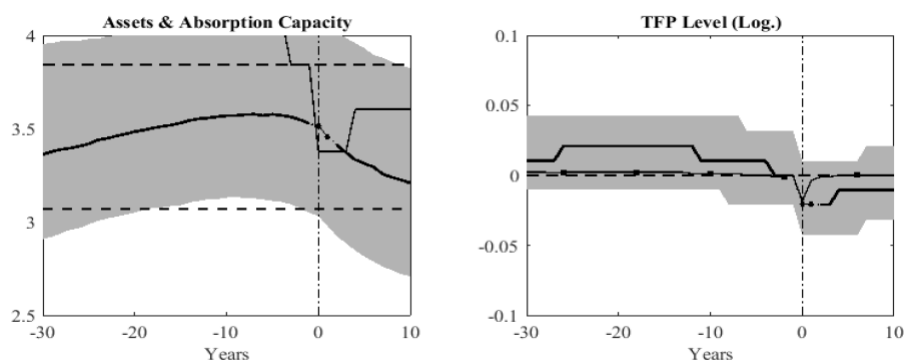Figure 3.9: Typical path to financial recessions in the Home country



Figure 3.10: Typical path to financial recessions in the Foreign country

Note that in the simulation, only the Foreign country is hit by the shock so that its banks' absorption capacity (thick plain line) as well as the endogenous dynamics of banks' assets (thin plain line) cross out with each other around the crises, while those in Home country are relatively stable.

Since two countries share the interbank market, the home country experiences financial recessions due to the contagion effect. Figure 3.11 shows the dynamics of the output and credit gaps around recessions in the home country. Compared to that in the closed economy, we see that the relative size and duration of financial recessions from simulations both increase in the world economy. Thus, they confirm that the model generates deeper and longer financial recessions than other recessions, which are associated with a
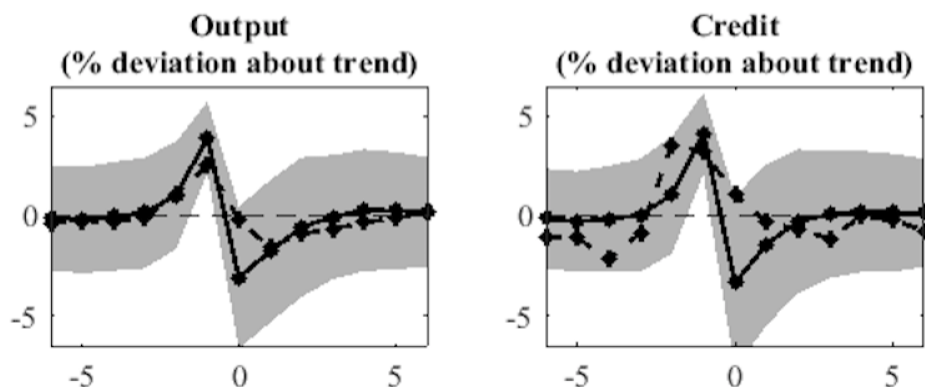
Figure 3.11: Dynamics of output and credit gaps around recessions in Home country

boom-bust cycle in the credit. However, our two-country model generates financial recessions with a frequency of 1.29 percent, nearly 50 percent lower than that in a closed economy as in Boissay et al. (2016). In other words, the very nature of a shared interbank market makes the economy less susceptible to negative productivity shocks, but the resulted crisis are more pronounced if they occur. The intuition is that, an internationally integrated interbank market endowed with a relatively larger "absorbing capacity" acts as a shield against negative shocks and fluctuations, which makes crisis less frequent. Rationally anticipating this, borrowers have more incentive to leverage and the gain from diversion increases as well. The interbank moral hazard issue potentially becomes more severe and implies larger macroeconomic contractions once induced by a crisis.

## 3.8   Concluding Remarks

In this chapter , we offered a two-country model where the countries are symmetric. Banks become heterogeneous by their private intermediation efficiency. This heterogeneity gives rise to an interbank market. However, due to the moral hazard problem along with private intermediation skills, less efficient banks may have an incentive to borrow in the interbank market and divert funds at a lower return asset. A borrowing capacity

constraint restricts these less efficient banks from diverting the funds. However, a lower corporate lending rate and interbank rate increases the incentive of these banks to divert funds and the interbank market responds by lowering the borrowing capacity. At some point, the interbank market goes to a freeze if the rates of return are sufficiently low.

A sequence of favorable productivity shocks lead to an expansion of credit. However, when the productivity growth goes down from an increasing trend, corporate lending rate goes down along with the demand. Counterparty risk goes up and the market stops lending to each other. When two countries are integrated through interbank lending, a market freeze in one country leads to a freeze in another country as well. Either two countries have an operating interbank market or both goes into a freeze. Excessive credit creation by one country triggers a two-country wide credit freeze. The countries may also go into a freeze from very low productivity in either of the countries.

In this chapter , we made some strong assumptions for tractability reason and presented a stylized model. Indeed, we left out some features of the banking sector that would deserve attention for future extensions. For example, we assume that households cannot lend the firms directly and firms cannot issue any equity or debt securities. Moreover, we assume that firms do not default on loans. Another possibility would be time or/and country varying return on storage technology. All these extensions are left for future research.

## 3.9   References

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# CHAPTER 4.   FUTURE WORK SUMMARY AND DISCUSSION

The study in chapter 2 can be extended in terms of the settlement after default. I assumed that the sovereign starts a new period with all previous debt forgiven by the lenders after a stochastic period of exclusion from the international credit market following a default. However, the sovereign is usually given extended period of repayment or granted debt cut during a settlement after default. It would be interesting to observe the default probabilities and bond prices after such inclusion of settlements as in Chatterjee and Eyigungor (2015). A more challenging work would be endogenizing the maturity structure of bonds where the sovereign not only chooses the level of short- and long-term bonds but also chooses the maturity of long-term bonds as an endogenous outcome. Arellano and Ramanarayanan (2012) and Hatchondo et al. (2016) study the multiple maturity structure but the sovereign does not choose the maturity structure endogenously. Therefore, this will be an interesting addition to the literature as well.

The study of chapter 3 can be extended in terms of the cost of diverting funds to the alternative storage technology by the low-skill banks. We assumed this storage technology is the same in both countries. It would be interesting to observe the interbank equilibrium with varying storage technology cost. Moreover, the return from the storage technology is also assumed the same in both countries. It would also be interesting to observe the interbak equilibrium with varying return from the storage technology. Boissay et al. (2016) has a closed economy model. We extended in various dimensions while keeping the parameters same in both countries.

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